NINTH INTERNATIONAL SYMPOSIUM ON

ORTHOGONAL POLYNOMIALS SPECIAL FUNCTIONS AND APPLICATIONS

JULY 2-6, 2007

CIRM, LUMINY, FRANCE

Conference Book

CONTENTS

SCIENTIFIC PROGRAM (july 2-6, 2007)

1 Introduction

The ninth International Symposium on Orthogonal Polynomials, Special Functions and Applications (OPSFA) is hosted at CIRM, which is an International Center for Mathematical Meetings, located on the campus of Luminy, close to Marseille, France.

Background and scope

The 9th International Symposium on Orthogonal Polynomials, Special Functions and Applications follows the European Conferences of Bar-le-Duc (France, 1984) opened by Jean Dieudonné, Segovia (Spain, 1986), Erice (Italy, 1990), Granada (Spain, 1991), Evian (France, 1992), Delft (Holland, 1994, in honour of Thomas jan Stieltjes (1856-1894)), Sevilla (Spain 1997), Patras (Greece, 1999, in honour of Theodore Chihara), Rome (Ostia, Italy, 2001), Copenhague (Denmark, 2003, in honour of Richard Askey) and Munich (Germany, 2005).

It covers the field of orthogonal polynomials, special functions and their applications to mathematics, physics, chemistry and biology.

The scientific program includes 17 plenary lectures of 60 minutes, 2 invited lectures of 30 minutes and 96 contributed lectures of 30 minutes in 5 parallel sessions.

Symposium Committees

The international scientific committee consists of:

- Francesco Altomare (Bari, Italy).
- Richard Askey (Madison, USA).
- Bernhard Beckermann (Lille, France).
- Christian Berg (Copenhagen, Denmark).
- Jesus Sanchez Dehesa (Granada, Spain).
- Mourad E. H. Ismail (Florida, USA).
- Valeriy Kalyagin (Nijni-Novgorod, Russia).
- Marek Kowalski (Warsaw, Poland).
- Alphonse Magnus (Louvain-la-Neuve, Belgium).
- Francisco Marcellan (Madrid, Spain).
- Herbert Stahl (Berlin, Germany).
- Galliano Valent (Paris, France).
- Walter Van Assche (Leuven, Belgium).

The local organizing committee consists of:

- Galliano Valent (Paris 7, France).
- Jacek Gilewicz (Toulon, France).
- Roland Triay (Aix-Marseille I, France).

PLEASE READ VERY CAREFULLY THIS SECTION

2 Practical information

Arrival and departure

The participants are supposed to arrive on sunday july 1st and leave on saturday july 7th. Accomodation at CIRM outside of this range is impossible, because each week a different Conference takes place.

Transportation

If you come by plane, train or car, read *carefully* the practical informations to reach CIRM on the web site:

http://www.cirm.univ-mrs.fr/web.ang/infos_pratiques/acces.php

From the Coaches Station (close to the Central Railway Station of Saint Charles: get out from the railway station by the main exit, turn to right and after 150 meters you will find it; from the airport you will arrive to it directly) there will be a coach of the Conference leaving at

17h 19h 21h 23h

The trip to Luminy is about half an hour.

Accomodation and meals

CIRM is a relatively small structure, which allows accomodation for at most 80 persons. So, we very much apologize, but we had to separate the participants into two groups:

1. People lodged at CIRM, and taking all their meals at CIRM's restaurant. The time schedule is:

breakfast: 7h → 9h lunch: 12h 30 dinner: 19h 30

2. People lodged at the students residence, in flat F. Only single rooms (brand new) are available with: toilets, shower, refrigerator but no television. *Bring just your own towels.* The participants lodged at the students residence will have to take their meals at the CROUS restaurants. Their opening hours are:

breakfast: 7h → 10h lunch: 11h 30 → 13h 45 dinner: 18h 30 → 20h 30

The prices for a meal are 5 − 7 euros. The CROUS staff promised us improved menus for that week.

Reception

Reception will be ensured on *sunday july 1st from 17h to 23h.*
On arrival everybody (people lodging either at CIRM or at the students residence, flat F) will get the key of his room and will be guided to it. A buffet will be served at CIRM's restaurant from 19h 30 to 22h 30 for people lodged at CIRM. Unfortunately we could not obtain this for all participants, due to the small size of CIRM's restaurant.

The CROUS restaurants are closed on sundays, so people lodged at CIRM arriving on the campus later than 22h 30 or people lodging at the students residence have to provide for their dinner before arriving at Luminy.

Registration and Information desk

There will be a desk in the Library building, for registration and information:

monday : 8h → 18h tuesday → thursday : 9h → 15h

If you need a signed document stating that you have attended the Conference, you can pick it up from CIRM secretary's office beginning on Thursday.

The accomodation of all participants has to be paid on arrival: monday or tuesday. For the participants lodged at CIRM they have to pay at CIRM reception desk. For the participants lodged at the students residence they should pay at the CROUS Office (in a building with a peculiar hexagonal structure). One can pay either by cash or by credit card and a bill will be given.
The opening hours are:

monday ⟶ friday : 9h ⟶ 12h 30 and 13h 30 ⟶ 16h

3 Social events

Welcome aperitive:

On tuesday it will be served for all participants and their accompanying persons at CIRM's restaurant, starting from 12h 30.

Conference tour and dinner:

On wednesday at 14h 30, buses will start from CIRM and will bring the participants to Cassis. They may visit this picturesque little harbour and possibly have a boat trip to the Calanques, seen from the sea, if the eastern wind is not too strong. At about 19h the buses will bring the participants to the Conference dinner that will take place at the "Moulin de Gémenos", not very far from Cassis. The buses will ensure the transportation of the participants back to Luminy at about 23h.

Walk in the Calanques:

A Calanque is just some steep rocky hill falling down into the sea, and most have a name. Bathing is possible, but the water is rather cold. Walking on this stony ground implies *good walking shoes* to have feet properly protected. The meeting point will be the University entrance (close to the little house of the watchman, 1 on the map) between 20h 30 and 20h 45. People with inappropriate shoes will not be accepted for the walk.

The walk to the belvedere is easy, and gives nice views on the Calanques of Sugiton (closest to CIRM) and of Morgiou.

4 Conference Proceedings

The plenary and invited talks will be refereed and published in a special issue of the "Journal of Computational and Applied Mathematics". The rules of this journal concerning publication of manuscripts will be followed and two referees will be asked to judge upon the suitability of the paper for being published. Further details can be found on the journal web site:

http://www.elsevier.com/wps/find/journaldescription.cws_home/505613/description#description

Manuscripts are expected to be of 5 pages (maximally 7) for invited talks and of of 10 pages (maximally 15) for plenary talks. A summary of already published results is not acceptable. The Proceedings have to be in one volume of the Journal, so 500 pages is a regular upper limit of the Proceedings.

The deadline for submission will be **October 31, 2007** and *no late submission will be accepted.* Authors must submit the PDF file of the manuscript to Jacek Gilewicz:

`gilewicz@cpt.univ-mrs.fr`

Only after the paper has been accepted an electronic version, with JCAM macros, should be submitted. The corresponding author will receive 50 reprints to share.

5 How to use CIRM's computers equipment

CIRM provides a network with cables and a wireless network. *Véronique Arnouil*, the secretary of the meetings will give the access codes to the organizer. Her room is located in the building called "Auditorium". If she is absent, ask for the codes to the reception. It is the organizer of the meeting who gives the codes to the participants.

If your have some other problems see *Marie-Goretti Dejean*, computer manager. Her room is located at the ground floor, inside the Library.

CIRM has 2 free access computers rooms in the library ; one at the ground floor, the other one, upstairs. You can use the computers connected on the mezzanine. Here are some answers to the FAQ **HOW TO...**

5.1 Use the free access computers

CIRM's computer services places at the proposal some linux PC (ubuntu), windows PC (XP), Mac OS X and sunray.

Login and password : ask them to your organizer for using the linux PC and the sunray. Use the account called "invité" with the mac and the windows PC.

5.2 Connect my laptop to the network

The connection is carried out by the DHCP server. You don't have to specifiy some DNS servers. If your laptop has one, it is better to delete it.
For network with cable your IP address must look like : 139.124.3.XXX
For the wireless, ask your organizer for the WEP key.
Your IP address must look like : 192.168.3.XXX

5.3 Use the wireless

(cf n **2** connect my laptop to the network). The wifi areas are :

1. All the rooms located in the Bastide and the Bastide's hall.
2. All the rooms by the patio.
3. All of the Library and of the Auditorium.

There are now several possibilities:

1. If you had already used the CIRM wireless network, omit the last WEP key and enter it again ; it could change with respect to the last time.

If you use windows, to type the new WEP key:

a) List the wireless networks.
b) Disconnect from the wireless network.
c) Type the new WEP key.

If you use Mac OS, to type the new WEP key:

a) Choose "other" in the airport menu (don't look at the CIRM network detected, it is the old one!)
b) Put the name and the new WEP key

2. If you never connected to CIRM wireless.

3. To connect: the WEP key is OK, launch your favourite browser. Enter the login and the password given by your organizer and validate.

5.4 Print

The name of the printer is "bibli". It is a

HP 4250 Black/White 2 sides printer, IP address 139.124.3.15

You will find it at the ground floor of the Library, in front of the wooden stairs and near the free access computers room. It is the default printer from all the free access computers.

To print from your laptop, you have to add this machine to your printers list.
If you use windows :

1. Click on “add a new printer” and choose “local printer”.
2. Use the port TCP/IP. Here put the IP address of the printer.
3. Choose “HP” and “Laserjet 4250”.

If you use Mac OS X:

1. Click on “add” in the “print center utility”.
2. Select the printer in the list. You can use “apple talk” or “LP”.

5.5 Use my USB stick

All the computers mount automatically USB keys.

Note for the sunray : the folder of your USB key is mount in:

/tmp/SUNWut/mnt/guest/NameOfYourKey

Just copy (cp) or move (mv) your selected file in the directory of guests (/other/guest).

5.6 Use the video appliances :

WARNING : you don’t have to disconnect all the VGA cables! If you have any problems, contact the computer service.

CIRM has 3 video appliances : 1 fixed, located in the Auditorium, and 2 portable.

5.6.1 How to connect your laptop in the conference room

1. Use the remote control to start the video appliance. On/Off button.
2. Connect your laptop with the VGA or DVI cable.
3. Commutate with the video appliance : For windows or linux: press Fn+ the function key represented by the monitor (it can be F5, F7 or F8...). Sometimes, with linux, you have to reboot.

   For Mac OS X : just pull down and up the screen of your laptop.

   To shut down the video appliance : use the remote control and press the button On/Off. When you see the question “éteindre ?” press the button again.

   If you want to start again the appliance, just wait for the red light turns to green.

5.6.2 Connect your laptop to a portable video appliance

1. To start the video appliance : press the button On/Off.
2. Connect the appliance to your laptop with the VGA or DVI cable.
3. Switch on the screen (Fn + function Key F5 or F7 or F8...)
4. To shut down the video appliance : press the On/Off button, one time, and the second time to confirm.
5. WAIT for the appliance finishes his cycle !! AFTER, you will able to unplug the battery. Thanks !

Recommendations

1. Do not shut down or reboot the computers.
2. Do not unplug the network cables or electric cables. The computer services can lend you a cable on if you need to. Give it back to the computer services or to the reception when you will leave.
3. Do not take off the VGA extension cable in the conference room.
4. Don’t waste your time to search how to use some appliances, ask for some help to the computer services. It will be useful for you and for us, anyway.

5.7 Electronic mail

The name of the SMTP server is *cirm.univ-mrs.fr*.

6 Plenary talks: time schedule

Notice:

- Plenary talks are scheduled for 50 minutes, followed by 5 minutes for questions and 5 minutes for rest.

- In view of the many participants to the Conference, the Plenary talks will take place in amphitheater number 8, of the nearby Université d'Aix-Marseille II, see the map (it's a 5 minutes walk from CIRM).

	Monday 2nd	Tuesday 3rd	Wednesday 4th	Thursday 5th	Friday 6th
Chairman:	*Askey*	*Kalyaguin*	*Koornwinder*	*Chihara*	*Koelink*
9h - 10h	Welcome	**Denissov**	**Lasser**	**Grunbaum**	**Beckermann**
10h - 11h	**Ramis**	**Plesniak**	**Kuijlaars**	**Golinskii**	**Marcellan**
Break					
11h30 - 12h30	**Magnus**	**Aptekarev**	**Martinez–Finkelshtein**	**Van Assche**	**Stahl**
Lunch		*aperitive*			
14H 30 - 15h30	**Killip**	**Ismail**	Tour to Cassis (15h)	**Simon**	**Cuyt/Berg**
Break					
16h -17h 30	Parallel	Sessions		Parallel	Sessions
Break					
18h -19h30	Parallel	Sessions		Parallel	Sessions
Dinner			Banquet (19h 30)		
20h45				*walk: belvedere*	
21h -22h	**Simon** (1)	**Simon** (2)			

7 Plenary talks: abstracts

Varying weights of orthogonality for family of polynomials with varying recurrence coefficients

ALEKSANDER APTEKAREV (Keldysh Institute, Russia)

We consider families of polynomials

$$\{P_{n,N}(\lambda)\}_{n=0}^{\infty} \quad : \quad \int P_{n,N}(\lambda)\lambda^k W_N(\lambda)\,d\lambda = 0, \qquad k = 0,\dots,n-1,$$

which are orthogonal with respect to a weight function, depending on the parameter $N \in \mathbb{N}$. The weight functions W_N are called *varying weights*. These polynomials satisfy a three-term recurrence relation

$$\lambda P_{n,N}(\lambda) = P_{n+1,N}(\lambda) + a_{n+1,N}P_{n,N}(\lambda) + b_{n,N}^2 P_{n-1,N}(\lambda),$$

$$P_{0,N} = 1, \qquad P_{-1,N} = 0,$$

and the coefficients $\{a_{n,N}, b_{n,N}\}_{n=0}^{\infty}, \quad b_{n,N} > 0$, which also depend on the parameter N, are called *varying recurrence coefficients*.

In this talk we discuss relations between varying weights and varying recurrence coefficients and we consider these relations from the point of view of spectral theory and potential theory. In particular we focus on the existence of the limits $\lim_{N\to\infty} W_N(\lambda)^{1/N} =: W(\lambda) =: e^{-2Q(\lambda)}$ whenever the asymptotic behavior of the varying recurrence coefficients is given by

$$\lim_{\substack{n/N\to x\\ N\to\infty}} a_{n,N} =: a(x), \qquad \lim_{\substack{n/N\to x\\ N\to\infty}} b_{n,N} =: b(x).$$

Quantum Hilbert matrices and orthogonal polynomials

CHRISTIAN BERG (University of Copenhagen, Denmark)

The quantum integers are defined by

$$[n]_q = \frac{q^{n/2} - q^{-n/2}}{q^{1/2} - q^{-1/2}}, \quad n = 0, 1, \ldots, \tag{1}$$

where $q \in \mathbb{C} \setminus \{0\}$. If q is not a root of unity the quantum Hilbert matrix is given by

$$\mathcal{H}_n(q) = \left(\frac{1}{[l+j+1]_q}\right), \quad 0 \le l, j \le n, \quad n = 0, 1, \ldots. \tag{2}$$

When $|q| < 1$ it is the Hankel matrix corresponding to certain little q-Jacobi polynomials. This has been used in [1] to give expressions for the entries of the inverse quantum Hilbert matrix. The formulas hold by analytic continuation outside the roots of unity. The limiting case $q = 1$ gives a formula of Choi for the integer entries of the inverse Hilbert matrix.

For the special value $q = (1-\sqrt{5})/(1+\sqrt{5})$ the matrix $\mathcal{H}_n(q)$ is closely related to a Hankel matrix of reciprocal Fibonacci numbers called a Filbert matrix, see [2], [3]. The formulas also extend Ismail's generalized Fibonacci numbers studied in [4].

[1] J. Ellegaard Andersen, Christian Berg, "Quantum Hilbert matrices and orthogonal polynomials", `math.CA/0703546`.
[2] T. M. Richardson, "The Filbert matrix", *Fibonacci Quart.*, **39** 3 (2001) 268-275.
[3] C. Berg, "Fibonacci numbers and orthogonal polynomials", to appear in *J. Comput. Appl. Math.*, `math. NT/0609283`.
[4] M. E. H. Ismail, "One Parameter Generalizations of the Fibonacci and Lucas Numbers". Preprint, August 2006.

Hermite-Padé forms and smoothing the Gibbs phenomenon

BERNHARD BECKERMANN (Université de Lille, France)

Recently, Padé techniques have been proposed and analysed in order to smooth the Gibbs phenomenon for partial Fourier sums of smooth functions with jumps. Supposing that the jump location is known, Driscoll and Fornberg suggested to use Hemite-Padé forms at zero in order to improve convergence, by presenting very convincing numerical experiments.

Suppose that we know the first coefficients of the Taylor series f_2 at zero of a function analytic in the disk with just one logarithmic singularity at 1 (which is typical in applications), we choose $f_1(z) = \log(1-z)$

having the same singular behavior, and look for polynomials p_0, p_1 and p_2 such that the Hermite-Padé form $R = p_0 + f_1p_1 + f_2p_2$ has an order as high as possible at zero. Finally, the value $f_2(z)$ for $|z| = 1$ is approached by solving the relation $R(z) = 0$ for $f_2(z)$.

We present some error estimates for these approximants with built-in singularities for the case where the degree of p_0 is large compared to the degrees of p_1 and p_2. Here Nikishin systems f_1, f_2 will play a prominent role.

Remark: it would be helpful for the participants to attend Wielonski's invited talk, on thursday afternoon (room A1-2 starting at 7 pm), which is strongly related with this talk.

Towards reliable software for the evaluation of a class of special functions

ANNIE CUYT (University of Antwerpen, Belgium)

Special functions are pervasive in all fields of science. The most well-known application areas are in physics, engineering, chemistry, computer science and statistics. Because of their importance, several books and a large collection of papers have been devoted to the numerical computation of these functions. But up to this date, even environments such as Maple, Mathematica, MATLAB and libraries such as IMSL, CERN and NAG offer no routines for the reliable evaluation of special functions. Here the notion reliable indicates that, together with the function evaluation a guaranteed upper bound on the total error or, equivalently, an enclosure for the exact result, is computed.

We point out how limit-periodic continued fraction representations of these functions can be helpful in this respect. The newly developed (and implemented) scalable precision technique is mainly based on the use of sharpened a priori truncation error and round-off error upper bounds for real continued fraction representations of special functions of a real variable. The implementation is reliable in the sense that it returns a sharp interval enclosure for the requested function evaluation, at the same cost as the evaluation.

The continuous analogs of polynomials orthogonal on the unit circle: some old and new results

SERGUEI DENISSOV (University of Madison, USA)

I will give an account of some new developments in the theory of Krein systems- the first order systems of differential equations that give rise to continuous analogs of polynomials orthogonal on the unit circle.

Contractions with a rank-one non-unitary part and truncated CMV matrices

LEONID GOLINSKII (Institute for Low Temperature Physics, Ukrain)

The main issue we address in the talk is a new model for completely non-unitary contractions with a rank-one non-unitary part acting in the Hilbert space of dimension $N \leq \infty$. This model complements nicely the well-known models of Sz.-Nagy-Foias and Livsic. We show that each such operator is unitarily equivalent to some truncated CMV matrix obtained from the complete CMV matrix by deleting the first row and column, and acting in ℓ^2 $(\mathbb{C}^N)$. This result can be viewed as a non-unitary version of the famous characterization of unitary operators with simple spectrum due to Cantero, Moral and Velázquez as well as an analog for contraction operators of the result by Arlinskii and Tsekanovskii concerning dissipative non self-adjoint operators with a rank-one imaginary part. We develop direct and inverse spectral analysis for finite and semi-infinite truncated CMV matrices. In particular, we study the problem of reconstruction of such matrices from their spectra or the mixed spectral data. In this part our results are closely related to the results of Hochstadt and Gesztesy-Simon for finite self-adjoint Jacobi matrices.

Matrix orthogonal polynomials, differential equations and applications

ALBERTO GRUNBAUM (University of California, USA)

The theory of matrix orthogonal polynomials, started by M G Krein in 1949, suffers from a scarcity of explicit nontrivial examples. This situation is starting to change as a result of different efforts stemming from various directions. The unifying theme in these new examples is that the family of orthogonal polynomials should be the common eigenfunctions of some differential operator (this property gives many of the familiar examples in the scalar case). I will give an overview of the results of these different searches, making special emphasis on the differences between the scalar and the matrix case. The results so far give rise to a number of mathematical problems with a much more interesting answer than the corresponding ones in the scalar case. If time allows it, I would like to describe some possible applications of these new families of matrix valued orthogonal polynomials.

Plancherel-Rotach asymptotics for q-orthogonal polynomials

MOURAD ISMAIL (University of Central Florida, USA)

We show that the q-orthogonal polynomials with unbounded recursion coefficients exhibit new types of asymptotics in the oscillatory range and around the largest zeros. The asymptotics in the complex plane will also be discussed. Some applications will be mentioned.

Random orthogonal polynomials and random matrices

ROWAN KILLIP (University of California, USA)

I will survey an approach to questions in random matrix theory via random orthogonal polynomials. This approach was discovered by Hale Trotter in the context of the classical ensembles and more recently has been extended to treat arbitrary (inverse) temperatures β.

Orthogonal polynomial ensembles and their limits

ARNO KUIJLAARS (Katholieke University of Leuven, Belgium)

Orthogonal polynomial ensembles are special random point sets on the real line arising for example from eigenvalues of unitary random matrices. The correlation functions have determinantal structure with a kernel that is the reproducing kernel for orthogonal polynomials on the real line, also known as the Christoffel-Darboux kernel.

As the degree tends to infinity, and after suitable rescaling, universal limiting kernels arise. In typical situations this leads to the sine kernel and the Airy kernel. In critical situations more complicated kernels arise that are related to Painleve transcendents.

Linearization coefficients of products of orthogonal polynomials and properties of Banach algebras defined by these coefficients

RUPERT LASSER (GSF-RCTUM Center of Mathematics, Germany)

We show how the positivity (or uniform boundedness) of linearization coefficients of products of orthogonal polynomials lead to several Banach algebras on the nonnegative integers respectively on the support of the orthogonalization measure. We investigate how growth conditions of these coefficients determine properties of these algebras. In particular, we present results on symmetry, spectral synthesis and amenability of these Banach algebras.

Rational interpolation to solutions of Riccati difference equations on elliptic lattices

ALPHONSE MAGNUS (Université Catholique de Louvain, Belgium)

An elliptic lattice, or grid, $\{x_0, x_1, \ldots\}$ is built with the help of a biquadratic curve $F(x, y) = \sum_{i=0}^{2} \sum_{j=0}^{2} c_{i,j} x^i y^j$ by the following rules:

1. let y_n and y_{n+1} be the two $y-$roots of $F(x_n, y) = 0$,
2. then, x_{n+1} is found as the remaining $x-$root of $F(x, y_{n+1}) = 0$.

There is also a direct symmetric biquadratic relation $E(x_n, x_{n+1}) = 0$, see [1].

Numerators and denominators of rational interpolants on such lattices satisfy interesting difference equations when the interpolated function f itself satisfies a Riccati difference equation on the same lattice:

$$a(x_n)\frac{f(y_{n+1}) - f(y_n)}{y_{n+1} - y_n} = b(x_n)f(y_n)f(y_{n+1}) + c(x_n)(f(y_n) + f(y_{n+1})) + d(x_n),$$

where a, b, c, and d are polynomials.

[1] V. P. Spiridonov and A. S. Zhedanov: "Elliptic grids, rational functions, and the Padé interpolation", *The Ramanujan Journal* **13**, 1-3 (2007) 285-310.

Linear combinations of orthogonal polynomials: orthogonality and zeros

FRANCISCO MARCELLAN (Universidad Carlos III, Spain)

We analyze some analytic properties of linear combinations of a sequence of monic polynomials (P_n) orthogonal with respect to a linear functional defined in the linear space of polynomials with real coefficients. We focus our attention in the study of necessary and sufficient conditions for the orthogonality of the sequence of monic polynomials (Q_n) given by

$$Q_n = P_n + a_1 P_{n-1} + ... + a_k P_{n-k}, \quad a_k \neq 0.$$

The cases $k = 1, 2$ are studied with a particular emphasis. On the other hand, we analyze the distribution of zeros of the sequence (Q_n) as well as their location in terms of the zeros of the sequence (P_n). Some extensions of this problem for non standard orthogonal polynomial sequences will be considered.

Information measures of orthogonal polynomials

ANDREI MARTINEZ-FINKELSHTEIN (Universidad de Almería, Spain)

The entropy of orthogonal polynomials proved to be a very interesting object, both because of its applications in mathematical physics and its deep connections with the analytic theory of special functions. It poses two major challenges: efficient computation and asymptotic behavior. In both fields interesting results have been obtained through the last few years, and a brief review is one of the goals of this talk. On the other hand, new information measures (such as Fisher and Renyi information, discrete entropy and others) can also be studied in relation with orthogonal polynomials. A revision of some new facts, as well as of a number of open problems, will be the other aim of the talk.

Jackson property of compact sets in $\mathbb{R}^N$

WIESLAW PLESNIAK (Universytet Jagielloński, Poland)

Let E be a compact subset of the space $\mathbb{R}^N$ such that $E = \overline{\mathrm{int\,E}}$. We shall say that E admits *Jackson's inequality* $(\mathcal{J})$ if for each $k = 0, 1, \dots$ there exist a positive constant C_k and a positive integer m_k such that for all $f \in C^\infty_{\mathrm{int}}(E)$ and all $n > k$ we have

$$(\mathcal{J}): \qquad n^k \mathrm{dist}_E(f, \mathcal{P}_n) \le C_k \, ||f||_{E,m_k}$$

where

$$\|f\|_{E,m} := \sum_{|\alpha| \le m} \sup_{x \in E} |D^\alpha f(x)|$$

and

$$\mathrm{dist}_E(f, \mathcal{P}_n) := \inf\{\sup_{x \in E} |f(x) - p(x)| : p \in \mathcal{P}_n\}.$$

Here $C^\infty_{\mathrm{int}}(E)$ denotes the space of all $\mathcal{C}^\infty$ functions in $\mathrm{int}E$ which can be continuously extended together with all their partial derivatives to E and $\mathcal{P}_n$ is the space of all polynomials of degree at most n. By known multivariate versions of the classical Jackson theorem, every compact cube P in $\mathbb{R}^N$ admits Jackson's inequality with $m_k = k + 1$. The purpose of our talk is to deliver other examples of Jackson sets in $\mathbb{R}^N$. We shall show, in particular, that a finite union of disjoint Jackson compact sets in $\mathbb{R}^N$ is also a Jackson set and that this in general fails to hold for an infinite union of Jackson sets. We also give a characterization of Jackson sets in the family of Markov compact sets in $\mathbb{R}^N$ which together with a Bierstone result permits to show that Whitney regular compact subsets of $\mathbb{R}^N$ are Jackson.

Classification of linear meromorphic q-difference equations

JEAN-PIERRE RAMIS (Université Paul Sabatier, France)

There is a classical "dictionary" introduced by Riemann between hypergeometric differential equations and complex linear two-dimensional representations of a (non commutative) free group generated by two elements (it involves trigonometric functions and gamma function). Then a lot of properties of hypergeometric functions are encoded by linear algebra and combinatorics. An extension of this dictionary to arbitrary *regular singular* linear meromorphic equations, on the Riemann sphere, on one side, and finite dimensional linear representation of finitely generated free groups, on the other side, was conjectured by Hilbert. It is the famous Riemann-Hilbert problem, which was solved during the XX-th century. The dictionary in this case is the *Riemann correspondance.*

During the first half of XX-th century, G.D. Birkhoff began to work on a large generalization of Riemann-Hilbert problem and published a series of deep papers on this topic. His aim was to skip the restriction "regular singular" (i. e. to allow irregular singularities, using *Stokes phenomena*) and to deal with "parent equations" as difference and q-difference equations.

I will explain how Birkhoff program was achieved in the differential case (between 1980 and 2000) and nearly achieved (between 1990 and today) in the q-difference case (it remains a work in progress in the difference case, with partial results). It involves the study of families of divergent series and resummation theories. The equations are classified by the finite dimensional representations of some new "fundamental groups". This classification is strongly related to differential and q-difference Galois theory and it can be used to solve direct and inverse problems in Galois theory. In the differential confluent hypergeometric case there are nice relations between our new fundamental group and special functions (as Kummer, Whittaker, Bessel, Airy functions...) and classical orthogonal polynomials (Laguerre and Hermite).

Fuchsian Groups and the Spectral Theory of Finite Gap Jacobi Matrices or Peherstorfer-Sodin-Yuditskii meet Killip-Simon

BARRY SIMON (Caltech, USA)

This is a report on joint work in progress with Jacob Christiansen and Maxim Zinchenko. After reviewing the non-local sum rules method of implementing the Killip-Simon results for perturbations of the free Jacobi matrix, I will describe the work of Sodin-Yuditskii and Perherstorfer-Yuditskii that relate spectral theory for finite gap Jacobi matrices to the study of suitable character automorphic functions under a certain Fuchsian group. I will then turn to new results: a non-local step-by-step sum rule in this setting and the other half of a Szego type theorem to the half proven by Widom, Aptakarev and Perherstorfer-Yuditskii.

Every so often, a paper appears that so changes our outlook on a problem that has already been heavily studied that it can only be called an earthquake. In the past six months two such papers have appeared (one with follow-ups). Barry Simon will present their results at OPSFA 2007 in two extra Lectures:

Monday (9 to 10 pm) in Auditorium:

(1) Lubinsky Earthquake: A Revolution in Universality and OP Zeros

Universality for the behavior of the off-diagonal CD kernel has been known in certain situations with analytic weights. Doron Lubinsky found a beautiful and extremely simple approach that more importantly requires only that the weight be continuous and bounded away from zero.

Lubinsky and Eli Levin then applied this to get clock behavior of the zeros in very great generality. (It should be mentioned that Freud at the end of his book realized that what we now call universality implies clock behavior.)

I will review what is needed from the Stahl-Totik regularity theory and present these ideas of Lubinsky and Levin-Lubinsky. If time allows, I will mention some extensions of Findley, Simon, and Totik and some other results on CD kernels.

The relevant preprints can be found at:

http://www.math.gatech.edu/ lubinsky/SelectedPapers.html

especially papers 199, 206.

Tuesday (9 to 10 pm) in Auditorium:

(2) Remling Earthquake: A Revolution in AC Spectrum

Understanding in terms of Jacobi parameters or Verblunsky coefficients when a measure has an ac part and where that part lives has been a major theme of spectral theory for at least 50 years. Examples are the Kato-Birman theory, Kotani theory, the Denisov-Rakhmanov theorem, and results on sparse potentials. Christian Remling recently posted a preprint that totally modifies our views of these problems.

Given a Jacobi matrix, we look at the two-sided Jacobi matrices that are limits at infinity. Last-Simon proved that such limits have spectrum contained in the essential spectrum of J and that on the other hand, the ac spectrum of J must be contained in the ac spectrum of the limits. What Remling has proven (using, in part, ideas of Breimesser- Pearson) is that this ac spectrum in the limits must be reflectionless.

I will explain his result, show how it implies the Denisov-Rahkmanov theorem in a few lines (without the need to go to the circle), and then pass to his striking new results including Denisov-Rakhmanov for general finite gap sets, his results on sparse potentials, and his results that any Jacobi matrix, where the a's and b's are chosen from a finite set which has ac. spectrum, is eventually periodic!

The relevant preprints can be found at:

http://arxiv.org/abs/0706.1101

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Rational Approximation on $(-\infty, 0]$

HERBERT STAHL (TFH Berlin, Germany)

We consider rational best approximants $r_n^* = r_n^*(f, (-\infty, 0]; \cdot) \in \mathcal{R}_{n,n}$ on $-(\infty, 0]$ to functions of the form

$$f = u_0 + u_1 \exp \tag{3}$$

with u_0, u_1 given rational functions.

Starting point of our investigations has been the solution of the $'1/9'$ problem by Gonchar & Rakhmanov in 1986, which deals with the uniform rational approximation of the exponential function on $(-\infty, 0]$, and it will also be the starting point for our talk. The renewed interest and the extension of the investigation to functions of type (3) is motivated by applications in numerical analysis, where exponential integrators demand good rational approximation of the exponential function and the related $'\varphi$ functions$'$, which are special cases of the functions of type (3).

It will be proved (not unexpectedly) that for functions of type (3), we have

$$\lim_{n\to\infty} \|f - r_n^*(f, (-\infty, 0]; \cdot)\|_{(-\infty,0]}^{1/n} = \frac{1}{9.28903\ldots} = H, \tag{4}$$

where H is Halven's constant that already came to fame with the $'1/9'$ problem. The convergence in (4) is extended to Hankel contours in $\overline{\mathbb{C}}\backslash(-\infty, 0]$, and it is shown that typical asymptotic properties of the best approximants $r_n^*(f, (-\infty, 0]; \cdot)$ hold already for nearly best approximants.

In one direction the analysis rather closely follows the path taken in Gonchar & Rakhmanov's research, here weighted orthogonal polynomials with respect to a Non-Hermitian orthogonality relation play a key role. In the reverse direction, new ways are explored and followed, and there some interesting potential-theoretic conclusions are brought into play.

If time allows, the algorithmic side of the problem will also be addressed. Here, again, orthogonal polynomials will play a leading role.

Hermite-Padé approximation to two functions with separated pairs of branch points

WALTER VAN ASSCHE (Katholieke Univ. Leuven, Belgium)

This talk is based on joint work with Alexander Aptekarev and Arno Kuijlaars. Let f_1, f_2 be functions of the form

$$f_i(z) = \int_{a_i}^{b_i} \frac{w_i(x)}{z-x}(x-a_i)^{\alpha_i}(x-b_i)^{\beta_i}\,dx$$

where $\alpha_i, \beta_i > -1$, $a_i, b_i \in \mathbb{C}$ and w_i, $1/w_i$ are both analytic in a domain Ω_i containing the branch points a_i, b_i. We assume the asymptotic expansion of f_i is known around $z = \infty$. These functions are well defined near infinity but depend on the path of integration $\Gamma_i \subset \Omega_i$ connecting a_i and b_i. We are interested in the simultaneous rational approximation of both function f_1, f_2 by Hermite-Padé approximation:

$$\begin{aligned} P_{n,m}(z) f_1(z) - Q_{n,m}(z) &= \mathcal{O}(1/z^{n+1}), \\ P_{n,m}(z) f_2(z) - R_{n,m}(z) &= \mathcal{O}(1/z^{m+1}), \end{aligned}$$

as $z \to \infty$. The rational functions have a common denominator and $Q_{n,m}/P_{n,m}$ is a rational approximation to some f_1 and $R_{n,m}/P_{n,m}$ is a rational approximation to some f_2. We show that Hermite-Padé approximation gives an approximation to these functions for particular branch cuts Γ_1, Γ_2. The geometry of the problem is determined by the position of the two pairs of branch points (a_1, b_1) and (a_2, b_2) and depends on a three-sheeted Riemann surface. The genus of this surface can range from 0 to 2. In this talk we only consider the case of genus zero. We use a Riemann-Hilbert problem for 3×3 matrices to obtain the asymptotic behavior of the common denominator $P_{n,m}$ when $n = m$ tends to infinity, from which the asymptotic zero distribution follows. We also obtain the convergence behavior for this Hermite-Padé approximation problem in the complex plane.

WARNING: SINCE THE CONFERENCE BOOK PRINTING, SOME CHANGES WERE NECESSARY IN THE TIME SCHEDULE OF THE PARALLEL SESSIONS. THE ON LINE VERSION IS THEREFORE THE MORE ACTUAL AND MAY CHANGE UP TO THE 28th OF JUNE.

8 Contributed talks: time schedule

Contributed talks are scheduled for 25 minutes (talk + questions). The remaining 5 minutes may be useful for changing room and for the next speaker to connect (if any) its laptop or USB stick and check that everything works...

Because of the very tight program we ask everybody to be punctual and to follow the schedule. In particular the role of the Chairmen will be of paramount importance in having the time schedule respected.

The contributed talks are organized in 5 parallel sessions, in the following lecture rooms:

Auditorium Library room A1-2 room A3 room A4

the last 3 lecture rooms are in the so-called “Annexe” part of CIRM.

Every room will be equipped with an overhead projector and a beamer. So either bring your transparencies or your laptop. If you just come with your USB stick, a PDF file is *strongly preferred.*

Monday

	Auditorium	Library	A1-2	A3	A4
Chairman	*Rahman*	*Golinskii*	*Beckermann*	*Berg*	*Ismail*
16h - 16h30	**Koornwinder**	Abey **Lopez**	**Berriochoa**	**Chiang**	**Abreu**
16h30 - 17h	**Ernst**	**Garza**	**Ben Cheikh**	**Koelink**	**Driver**
17h - 17h30	**Youssfi**	**Velazquez**	**Sanchez Moreno**	**Pedersen**	**Huber**
Break					
Chairman	*Plesniak*	*Marcellan*	*Martinez-F*	*Clarkson*	*Aptekarev*
17H30 - 18h	**Lal Sahab**	**Pijeira**	**Dai**	**Nijhoff**	**Illan**
18h30 -19h	**Révész**	**Dueñas-Ruiz**	**Miña-Diaz**	**de Bie**	**Yudistskii**
19h -19h30	**Maslouhi**	**Dziri**	**Tibboel**		**Lapik**

Tuesday

	Auditorium	Library	A1-2	A3	A4
Chairman	*Koornwinder*	*Grunbaum*	*Magnus*	*Dehesa*	*Golinskii*
16h - 16h30	**Fernandez**	**Castro**	**Milovanovic**	**Ferreira**	**Bessis**
16h30 - 17h	**Piñar**	**Cotrim**	**Kupin**	**Jordaan**	**Gishe**
17h - 17h30	**Rodal Vila**	**Coffey**	**Pranić**	**Perez Sinusia**	**Wong**
Break					
Chairman	*Koelink*	*Stahl*	*Denissov*	*Van Doorn*	*Kuijlaars*
17H30 - 18h	**Lewanowicz**	**Tokarzewski**	**Laforgia**	**Boukhemis**	**Clarkson**
18h30 -19h	**Loureiro**	**Yattselev**	**Dimitrov**	**Griffiths**	**Foulquie**
19h -19h30	**Rahman**	**Dominici**		Moritz **Simon**	**Johnston**

Thursday

	Auditorium	Library	A1-2	A3	A4
Chairman	*Simon*	*Aptekarev*	*Ismail*	*Kuijlaars*	*Van Assche*
16h - 16h30	**Christiansen**	**Lopez Lagomasino**	**Bouzeffour**	**Suarez Rodriguez**	**Terwilliger**
16h30 - 17h	**Zinchenko**	**Szafraniec**	**Bettaibi**	**Varona**	**Zudilin**
17h - 17h30	**Cantero**	**Bracciali**	**Van Doorn**	**Zarzo**	**Leopold**
Break					
Chairman	*Magnus*	*Askey*	*Plesniak*	*Lasser*	*Killip*
17H30 - 18h	**Branquinho**	Jose **Lopez**	**Kalyagin**	**Assal**	**de Schepper**
18h30 -19h	**Ronveaux**	**Llevens**	**Mantica**	**Haouam**	**Smet**
19h -19h30	**Bondarenko**	**Turaev**	**Wielonski**	**Koumandos**	**Hussain**

9 Contributed talks: Abstracts

Continuous and discrete orthogonal polynomials: a problem from OPSFA 2003

Daniel ABREU

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We will present a proof of the first conjecture made by Mourad Ismail in the open problems session of OPSFA 2003 in Copenhagen and published in the Proceedings of the conference: "M. E. H. Ismail, Problem 5. Orthogonality and Completeness, JCAM 178 (2005) 533-534.". There is a preprint of this at http://arxiv.org/pdf/math.CA/0601190. We will also make a brief discussion concerning natural questions that arise from this problem.

Functional analysis on the Laguerre hypergroups and applications

Miloud ASSAL

Dpartement de Mathématiques, Faculté des Sciences de Bizerte, Jarzouna - Bizerte - 7021 Tunisia.

In this talk we study generalized Besov and Sobolev type spaces associated with Laguerre Operators. We give different characterizations of these spaces in terms of generalized convolution with a kind of smooth functions and by means of generalized translation operators. Also a discrete norm is given to obtain more general properties on these spaces.

Askey-Scheme and d-Orthogonality

Youssef BEN CHEICK, Imed LAMIRI and Aabelwaheb OUNI

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Let $\mathcal{P}$ be the vector space of polynomials with coefficients in $\mathbb{C}$ and let $\mathcal{P}'$ be its dual. We denote by $\langle u, f\rangle$ the effect of the functional $u \in \mathcal{P}'$ on the polynomial $f \in \mathcal{P}$. Let $\{P_n\}_{n\geq 0}$ be a sequence of polynomials in $\mathcal{P}$ such that $\deg P_n(x) = n$ for all n and let d be an arbitrary positive integer. The polynomial sequence $\{P_n\}_{n\geq 0}$ is called a d-orthogonal polynomial sequence (d-OPS, for shorter) with respect to a d-dimensional functional $\mathcal{U} = {}^t(u_0, \cdots, u_{d-1})$ if it fulfills:

$$\begin{cases} \langle u_k, P_m P_n\rangle = 0 \ , \quad m > dn + k \ , n \geq 0, \\ \langle u_k, P_n P_{dn+k}\rangle \neq 0 \ , \ n \geq 0, \end{cases}$$

for each integer k belonging to $\{0, 1, \ldots, d-1\}$. For $d = 1$, we recognize the well-known notion of orthogonality.
In this talk, we define a general class of generalized hypergeometric polynomials $\mathcal{A}$ containing all the generalized hypergeometric representations of the OPSs in the Askey-scheme. Then we solve the characterization Problem **P** which consists to find all d-OPSs having generalized hypergeometric representations in $\mathcal{A}$.
For $d = 1$, we obtain the following characterization theorem:
The only OPSs having generalized hypergeometric representations in $\mathcal{A}$ are the thirteen PSs given by Askey-scheme and the Bessel polynomials.
The general solution of **P** allows us to construct a "*d-Askey-scheme*" containing d-orthogonal generalizations of all OPSs in Askey-scheme. The case $d = 2$, will be discussed for illustration.

Characterizing curves satisfying the Gauss-Christoffel-Darboux theorem

Elias BERRIOCHOA

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We study the converse of the Gauss theorem for quadrature formulas. Indeed we characterize the support of the measures having quadrature formulas with the exactness given in the Gauss theorem.

Padé Approximations of purely noisy Taylor Series and open questions

Daniel BESSIS and Luca PEROTTI

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We analyze the statistical distribution of the poles and zeros of Padé Approximations built on purely noisy Taylor series coefficients. How much is the final statistical distribution of the poles and zeros influenced by the different classes of input, such as blank, gaussian and pink noisy Taylor coefficients, is discussed among other open questions.

On Some Inequalities of Uncertainty Principles type in Quantum Calculus

Ahmed FITOUHI, Néji BETTAIBI, Rym BETTAIEB and Wafa BINOUS

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The aim of this paper is to generalize the q-Heisenberg uncertainty principles studied in arXiv `math.QA/0602658`, to state local uncertainty principles for the q-Fourier-cosine, the q-Fourier-sine and the q-Bessel-Fourier transforms, then to give an uncertainty principle of Heisenberg type for the q-Bessel-Fourier transform.

Special functions in Clifford-superanalysis

Hendrik de BIE and F. SOMMEN

Ghent University, Department of Mathematical Analysis, Galglaan 2, 9000 Ghent, Belgium.

We start our talk with a brief introduction to Clifford analysis on superspace (a space with both commuting and anti-commuting co-ordinates, see [1]. Next we generalize the Clifford-Hermite and the Clifford-Gegenbauer polynomials (see [2]) to this new framework in a merely symbolic way. More precisely, our approach does not a priori need an integration theory in superspace. Furthermore we prove a lot of basic properties, such as orthogonality relations, Rodrigues formulae, differential equations and recursion formulae.

We end our talk by discussing a physical application of these special functions, thus also obtaining an interpretation for the super-dimension, a numerical parameter we introduced to describe our superspaces.

[1] De Bie, H., Sommen, F., "A Clifford analysis approach to superspace", accepted for publication in Annals of Physics.
[2] Delanghe, R., Sommen, F., Souček, V., "Clifford algebra and spinor-valued functions", Kluwer Academic Publishers, Dordrecht, 1992.

New asymptotic estimates for spherical designs

Andriy BONDARENKO and Maryna S. VIAZOVSKA

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Let S^n be the unit sphere in R^{n+1}. The set of vectors $\vec{x}_1, \ldots, \vec{x}_N \in S^n$ is called a *spherical t-design* if

$$\frac{1}{meas\, S^n} \int_{S^n} p(\vec{x}) d\vec{x} = \frac{1}{N} \sum_{i=1}^{N} p(\vec{x}_i)$$

for all algebraic polynomials in $n+1$ variables and of degree at most t. Let $N = N(n,t)$ be a minimal cardinality of spherical t-designs. Korevaar and Meyers have proved the upper bounds $N(n,t) \leq C(n) t^{(n^2+n)/2}$, for fixed $n \in \mathbb{N}$ and $t \to +\infty$. Our main result is

Theorem 1. Let a_n be the sequence defined by

$$a_1 = 1, \quad a_{2n} = a_{2n-1} + 2n, \quad a_{2n-1} = 2a_{n-1} + n, \quad n \in \mathbb{N}.$$

Then, for all $n,t \in \mathbb{N}$ we have $N(n,t) \leq C(n) t^{a_n}$, where $C(n)$ is a constant depending only on n.

Corollary 1. For all $n,t \in \mathbb{N}$, $n \neq 1$, we have

$$N(n,t) \leq C(n) t^{\frac{3}{2} n \log_2 n}.$$

On the linear "generalized" birth and death processes

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We show that the linear "$2-$generalized" birth and death processes (i.e. the processes defined from two death rates and one birth rate) could be generated by the $2-$orthogonal polynomials sets. In particular, we give a characterization of these processes, when their related $2-$orthogonal polynomials are Sheffer-Meixner 's type.

Also, we show that in this particular case, it is possible to give an integral representation of the measures of orthogonality. In the general case, we give only the integal equations satisfied by the the measures of orthogonality.

Finally, we describe one model in enzymology, which involves "$2-$generalized" birth and death processes.

On the Askey-Wilson functions

Fethi BOUZEFFOUR

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An explicit expansion formula for the Askey-Wilson functions in terms of the Askey-Wilson polynomials is proved. Applications to finite continuous Askey-Wilson transform are given.

Zeros of Gegenbauer-Sobolev orthogonal polynomials

Cleonice F. BRACCIALI, Eliana X.L. de ANDRADE and A. SRI RANGA

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Iserles et al. [1] introduced the concepts of coherent pairs and symmetrically coherent pairs of measures with the aim of obtaining Sobolev inner products with their respective orthogonal polynomials satisfying a particular type of recurrence relation. Groenevelt [2] considered the special Gegenbauer-Sobolev inner products, covering all possible types of coherent pairs, and proves certain interlacing properties of the zeros of the associated orthogonal polynomials. In this work we extend the results of Groenevelt, when the pair of measures in the Gegenbauer-Sobolev inner product no longer form a coherent pair.

[1] A. Iserles, P.E. Koch, S.P. Nørsett and J.M. Sanz-Serna, "On polynomials orthogonal with respect to certain Sobolev inner products", *J. Approx. Theory*, **65** (1991) 151–175.
[2] W.G.M. Groenevelt, "Zeros of Sobolev orthogonal polynomials of Gegenbauer type", *J. Approx. Theory*, **114** (2002) 115–140.

Second order differential equations in the theory of orthogonal polynomials on the unit circle

Maria das NEVES REBOCHO and Amilcar BRANQUINHO

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In this talk we present extensions to the OPUC of the results established by Hahn and Maroni concerning the Laguerre-Hahn class of orthogonal polynomials on the real line.

Given a hermitian linear functional u and the corresponding function F defined by $F(z) = \langle u_\xi, \frac{\xi+z}{\xi-z} \rangle$, (if u is positive definite then F is the Carathéodory function corresponding to u), the functional u (or F) is said to be Laguerre-Hahn if F satisfies a Riccati differential equation with polynomial coefficients $zAF' = BF^2+CF+D$. The corresponding sequence of orthogonal polynomials is called Laguerre-Hahn. The Laguerre-Hahn class of orthogonal polynomials on the unit circle includes the semi-classical and Laguerre-Hahn affine orthogonal polynomials on the unit circle.

Let u be a Laguerre-Hahn regular functional and $\{\phi_n\}, \{\Omega_n\}$ and $\{Q_n\}$ be the sequences of the corresponding orthogonal polynomials, associated polynomials of the second kind and functions of the second kind, respectively. We give two characterizations of Laguerre-Hahn orthogonal polynomials on the unit circle:
a) in terms of a first order structure relation for $\{\phi_n\}, \{\Omega_n\}$ and $\{Q_n\}$;
b) in terms of a second order differential equation for the vectors $[\phi_n \ \ \Omega_n]^T$ and for $\{Q_n\}$.

Poisson brackets of orthogonal polynomials

María José CANTERO and Barry SIMON

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It is known since the discoveries of Flaschka and Moser concerning the finite Toda lattice as a completely integrable system that the finite Jacobi matrices of fixed trace support a natural symplectic form.

A natural question that arises is the study of the analogous one for CMV matrices, that is, to understand the connection of the symplectic structure of Nenciu and Simon to a completely integrable system. For the standard symplectic forms on Jacobi and CMV matrices we compute Poisson brackets of OPRL and OPUC, and relate these to other basic Poisson brackets and to Jacobians of basic change of variable. This work is related to work of Golinskii and of Gekhtman-Nenciu.

Matrix valued polynomials and the noncommutative bispectral problem

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The bispectral problem was first considered in the scalar case by J. J. Duistermaat and A. Grünbaum (see [2]) when one differential operator has order two and the other differential operator has arbitrary order. At the heart of this lies the need to solve some complicated nonlinear equations called *ad-conditions.* This approach was later used to rederive a classical result of Bochner for scalar orthogonal polynomials when both operators, a difference and a differential one, have order two. When the *functions* in question are matrix valued a first exploration of the power of this method is carried out in by A. Grünbaum and P. Iliev (see [3]).

Here we consider the case of matrix valued *polynomials* and treat mainly the case when both operators, a difference and a differential one, are of order one (see [1]). Once again at the heart of our approach is the solution of the ad-conditions which now becomes a difficult matter in view of the noncommutative nature of the coefficient matrices. The fact that we have a two term recurrence relation takes us away from the realm of orthogonal polynomials, but the methods given here should play an important role in classifying all families of matrix orthogonal polynomials satisfying first order differential equations.

[1] M. M. Castro and F. A. Grünbaum, " The noncommutative bispectral problem for operators of order one", *Constr. Approx.* (2007) to appear
[2] J. J. Duistermaat and F. A. Grünbaum, "Differential equations in the spectral parameter", *Comm. Math. Phys.*, **103** (1986), 177–240.
[3] F. A. Grünbaum and P. Iliev, "A noncommutative version of the bispectral problem", *J. of Computational and Appl. Math.*, **161** (2003), 99-118.

On the Nevanlinna order of Lommel and Struve functions and complex differential equations

Edmund CHIANG

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In an earlier joint work with M. Ismail [Canadian J. Math. **58** (2006), 257-287], we investigated a class of homogeneous ordinary differential equations in the complex plane with Morse potential that can admit entire solutions with "small" Nevanlinna order of zeros in $\mathbb{C}$ if and only it can be solved in terms of Bessel polynomials. We continue our study into a class of non-homogeneous ordinary differential equations in the complex plane and show it can admit "sub-normal solution" if and only if the solution can be written in terms of a composition of degenerated forms of Lommel or Struve functions and exponential function. New identities and properties of the Lommel and the Struve functions are established (Joint work with K. W. Yu).

A Szegő-type theorem for finite gap Jacobi matrices

Jacob S. CHRISTIANSEN

Jacob Christiansen, Caltech, Mathematics 253-37, Pasadena, CA 91125.

In the talk I will present parts of recent joint work with B. Simon and M. Zinchenko on finite gap Jacobi matrices. In particular, I will focus on a Szegő-type theorem and show how this result can be obtained using a non-local step-by-step sum rule.

Asymptotics and Connection Formulae for the Painlevé Equations

Peter CLARKSON

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The six Painlevé equations were first discovered around the beginning of the twentieth century by Painlevé, Gambier and their colleagues in an investigation of nonlinear second-order ordinary differential equations. Recently there has been considerable interest in the Painlevé equations primarily due to the fact that they arise as reductions of the soliton equations which solvable by inverse scattering. Although first discovered from strictly mathematical considerations, the Painlevé equations have arisen in a variety of important physical applications including statistical mechanics, random matrices, plasma physics, nonlinear waves, quantum gravity, quantum field theory, general relativity, nonlinear optics and fibre optics. Further the Painlevé equations may be thought of a nonlinear analogues of the classical special functions.

In this talk I shall discuss asymptotics and connection formulae for the first, second and fourth Painlevé equations. In particular I shall exhibit some new monotonically increasing and monotonically decreasing solutions of the second Painlevé equation.

[1] P.A. Clarkson, Painlevé equations — nonlinear special functions, in “Orthogonal Polynomials and Special Functions: Computation and Application”, [Editors F Marcellàn and W van Assche], *Lect. Notes Math.*, vol. **1883**, Springer-Verlag, Berlin (2006) 331-411.
[2] A.S. Fokas, A.R. Its, A.A. Kapaev and V.Yu. Novokshenov, “Painlevé Transcendents: the Riemann-Hilbert approach”, *Math. Surv. Mono.*, vol. **128**, Amer. Math. Soc., Providence (2006).

The theta-Laguerre calculus formulation of the Li-Keiper constants

Mark COFFEY

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The Riemann hypothesis is equivalent to the nonnegativity of a sequence of real constants $\{\lambda_k\}_{k=1}^{\infty}$, that are certain logarithmic derivatives of the Riemann xi function evaluated at unity. We re-express these constants using the theta-Laguerre calculus. By using integral representations, we reformulate the coefficients $\{\lambda_k\}_{k=1}^{\infty}$ together with a closely related sequence $\{a_j\}_{j=0}^{\infty}$. We present a decomposition of the quantities a_j into superdominant and subdominant components and give an upper bound on the former and an asymptotic lower bound for the latter. We also describe an arithmetic formula for the Li-Keiper constants and its relation to the theta-Laguerre calculus.

Algebraic Theory of Multiple Orthogonal Polynomial

Amlcar BRANQUINHO and Luis COTRIM

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In this talk we present an algebraic theory of multiple orthogonal polynomials. Our departure point is the three term recurrence relation, with matrix coefficients, satisfed by a sequence of vector multiple orthogonal polynomials. We give some characterization of multiple orthogonal polynomials including recurrence relations, extension of the Shohat-Favard theorem and of the Christoffel-Darboux formulas. A reinterpretation of the problems of Hermite-Padé approximation is presented.

Asymptotics of Discrete Orthogonal Polynomials and Riemann-Hilbert Problems

Dan DAI

Katholieke Univ. Leuven, Celestijnenlaan 200 B, 3001 Leuven, Belgium.

In this talk, we study the asymptotics of the Krawtchouk polynomials $K_n^N(z;p,q)$ as the degree n becomes large. Asymptotic expansions are obtained when the ratio of the parameters n/N tends to a limit $c \in (0,1)$ as $n \to \infty$. The results are globally valid in one or two regions in the complex z-plane depending on the values of c and p; in particular, they are valid in regions containing the interval on which these polynomials are orthogonal. Our method can also be applied to study the asymptotics of the Charlier polynomials $C_n^{(a)}(x)$.

Monotonicity of zeros of Jacobi polynomials

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Denote by $x_{n,k}(\alpha,\beta)$, $k=1,\ldots,n$, the zeros of the Jacobi polynomial $P_n^{(\alpha,\beta)}(x)$. It is well known that $x_{n,k}(\alpha,\beta)$ are increasing functions of β and decreasing functions of α. In this talk we discuss the question of how fast the functions $1-x_{n,k}(\alpha,\beta)$ decrease as β increases. We report a result that the products $t_{n,k}(\alpha,\beta) := f_n(\alpha,\beta)\,(1-x_{n,k}(\alpha,\beta))$, where $f_n(\alpha,\beta) = 2n^2 + 2n(\alpha+\beta+1) + (\alpha+1)(\beta+1)$, are already increasing functions of β. When $\alpha > -1$ is fixed, $f_n(\alpha,\beta)$ is the asymptotically extremal, with respect to n, function of β that forces the products $t_{n,k}(\alpha,\beta)$ to increase.

Polynomial solutions of differential-difference equations

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We investigate polynomial solutions of the first order differential-difference equation

$$P_{n+1}(x) = a_n(x)P_n(x) + b_n(x)P_n'(x),$$

where $a_n(x)$, $b_n(x)$ are polynomials of degree 1 and 2 respectively. We discuss possible orthogonal solutions and provide several examples.

We analyze the particular case corresponding to the Bell polynomials

$$P_{n+1}(x) = xP_n(x) + xP_n'(x),$$

asymptotically as $n \to \infty$ using a discrete version of the ray method. We obtain asymptotic approximations in the exponential, oscillatory and transition regions.

Polya frequency sequences and real zeros of some ${}_3F_2$ hypergeometric polynomials

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We prove that the zeros of some families of ${}_3F_2$ hypergeometric polynomials are all real and negative. Our approach uses the relationship between Polya frequency functions and the real zeros of polynomials linked with Polya frequency functions.

We also establish the asymptotic zero distribution of these zeros when the degree of the polynomials tends to infinity (Joint work with Andrei Martinez-Finkelshtein and Kerstin Jordaan).

Laguerre-Type orthogonal polynomials; electrostatic interpretation

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In this contribution we study the second order linear differential equation satisfied by polynomials orthogonal with respect to the linear functional

$$\langle \widetilde{\mu}, p \rangle = \int_0^{+\infty} p(x) x^\alpha e^{-x} dx + Mp(0)$$

where $\alpha > -1$, $M \in \mathbb{R}_+$, and p is a polynomial with real coefficients. We also find some results concerning the distribution of their zeros. Finally, an electrostatic interpretation of the zeros in terms of a logarithmic potential with an external field is presented.

Inversion formula for the generalized Riemann-Liouville transform and its dual associated with singular partial differential operators

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The generalized Riemann-Liouville transform and its dual transform associated with singular partial differential operator are defined and investigated. Here, from harmonic analysis for the related Fourier transform, some results are established. We also give inversion formula for these operators and a Plancherel theorem for the operator ${}^t\mathcal{R}$.

Jackson type estimates of q-monotone approximation

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Denote by $\Delta^{(q)}(Y)$, $q = 0, 1, 2$, three sets of all real continuous 2π-periodic functions f changing its sign (case $q = 0$)/monotonicity($q = 1$)/convexity ($q = 2$) at the fixed distinct points $y_i \in Y := \{y_i\}_{i\in\mathbb{Z}}$ (at least twice on $[-\pi, \pi)$). We constructed three trigonometric polynomials $T_{n,q} \in \Delta^{(q)}(Y)$ of order $\leq n-1$ such that if $f \in \Delta^{(q)}(Y)$ then

$$||f - T_{n,q}|| \leq c(Y)\,\omega_k(f, \pi/n), \quad n \in \mathbb{N}, \quad k = \begin{cases} 2, & \text{if} \quad q = 1, \\ 3, & \text{if} \quad q = 0, 2, \end{cases}$$

where $\omega_k\,(f, t)$ is the k-th modulus of continuity of f, $c(Y)$ is a constant depending only on Y and $||\cdot||$ is the max-norm. With grater k or with a constant independent of Y the respective estimates are wrong. The case $q = 0$ was proved by the author and J. Gilewicz, $q = 1$ − by the author and M.G. Pleshakov, $q = 2$ − by the student of the author V.D. Zalizko.

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q-Pascal and q-Bernoulli matrices, an umbral approach

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A q-analogue $H_{n,q} \in \mathbb{C}(q)$ of the Polya-Vein matrix is used to define the q-Pascal matrix. The Nalli-Ward-AlSalam (NWA) q-shift operator acting on polynomials is a commutative semigroup. The q-Cauchy-Vandermonde matrix generalizing Aceto-Trigiante is defined by the NWA q-shift operator.

A new formula for a q-Cauchy-Vandermonde determinant expressed as a product of q-Ward numbers is found. The matrix form of the q-derivatives of the q-Bernoulli polynomials can be expressed in terms of the $H_{n,q}$. With the help of a new kind of matrix multiplication, another q-analogue of Aceto-Trigiante is found. The q-Cauchy-Vandermonde matrix can be expressed in terms of the q-Bernoulli matrix. With the help of the Jackson-Hahn-Cigler (JHC) q-Bernoulli polynomials, the q-analogue of the Bernoulli complementary argument theorem is obtained. Analogous results for q-Euler polynomials are obtained.

A Hahn like characterization of bivariate classical orthogonal polynomials

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Classical orthogonal polynomials in two variables can be characterized as the polynomial solutions of a matrix second order partial differential equation involving matrix polynomial coefficients. In this work, we study classical orthogonal polynomials in two variables whose partial derivatives satisfy again a second order partial differential equation of the same type.

A complete study of the asymptotics relations and limits between the orthogonal polynomials of the Askey's table

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It has been shown that the fourth lower levels of the Askey table of hypergeometric orthogonal polynomials are connected by means of asymptotic relations. In this work we establish a complete asymptotic study between the five different levels. From these expansions, several limits between polynomials are derived. Some numerical experiments show the accuracy of the approximations and the accuracy in the approximation of the zeros of those polynomials (Work is in collaboration with José L. López, Pedro Pagola and Ester Pérez Sinusía).

[1] C. Ferreira, J.L. López and E. Mainar, Asymptotic relations in the Askey scheme for hypergeometric orthogonal polynomials, *Adv. in Appl. Math.*, **31**, 1 (2003), 61-85.
[2] R. Koekoek and R. F. Swarttouw, Askey scheme or hypergeometric orthogonal polynomials, *http://aw.twi.tudelft.nl/koekoek/askey* (1999).
[3] J.L. López and N. M. Temme, The Askey scheme for hypergeometric orthogonal polynomials viewed from asymptotic analysis, *J. Comp. Appl. Math.* **133** (2001) 623-633.

The Riemann-Hilbert analysis for multiple Jacobi polynomials

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Using the approach that multiple orthogonal polynomials are the solution of a Riemann Hilbert problem, we study the strong asymptotic behavior for multiple Jacobi polinomials for a Nikishin system of measures, that is let $\Delta_1 = [a_1, a_2]$ and $\Delta_2 = [b_1, b_2]$ be two intervals in the real line, such that $\Delta_1 \cap \Delta_2 = \emptyset$. For each $j = 1, 2$ we take a holomorphic function ρ_j on a neighborhood $\mathcal{V}(\Delta_j)$ of Δ_j. Given $\alpha_j > -1$ and $\beta_j > -1$, $j = 1, 2$, we define the measures σ_1 and σ_2 with their differential forms:

$$d\sigma_1(x) = (x - a_1)^{\alpha_1}(a_2 - x)^{\alpha_2}\rho_1(x)dx, \qquad x \in \Delta_1,$$

$$d\sigma_2(x) = (x - b_1)^{\beta_1}(b_2 - x)^{\beta_2}\rho_2(x)dx, \qquad x \in \Delta_2.$$

Let $S = (s_1, s_2) = \mathcal{N}(\sigma_1, \sigma_2)$ denote the Nikishin system that corresponds to (σ_1, σ_2) whose measures have the following differential forms

$$ds_1(x) = d\sigma_1(x) = w_1(x)dx, \quad ds_2(x) = \int_{\Delta_2} \frac{d\sigma_2(t)}{x - t} d\sigma_1(x) = \widehat{\sigma}_2(x)d\sigma_1(x) = w_2(x)dx, \qquad x \in \Delta_1.$$

Now we study the strong asymptotic of sequences of multiple orthogonal polynomials that satisfies the following orthogonality conditions

$$\int_{\Delta_1} x^\nu Q_{\mathbf{n}}(x)ds_j(x) = 0, \quad \nu = 0, \ldots, n_j - 1, \quad j = 1, 2.$$

Rational spectral transformations and measures on the unit circle

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We analyze some perturbation of a nontrivial positive measure supported on the unit circle. This perturbation is the inverse of the Christoffel transformation and is called the Geronimus transformation. We study the corresponding sequences of monic orthogonal polynomials as well as the connection between the associated Hessenberg matrices. Finally, we show an example of this kind of transformation.

Resultants of Chebyshev Polynomials

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Recently Dillcher and Stolarsky used algebraic methods to evaluate the resultant of two linear combinations of Chebyshev polynomials of the second kind. In here we give an alternative method of computing the same resultant and resultants of more general combinations of Chebyshev polynomials of the second kind. We also consider resultants of linear combinations of Chebyshev polynomials of the first kind.

Stochastic Processes with Orthogonal Polynomial Eigenfunctions

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Continuous time Markov Processes $\{X(t), t \geq 0\}$ with stationary distributions and Orthogonal Polynomials as eigenfunctions have characterizations in terms of extreme points in the class of processes. A natural way to consider general processes with fixed stationary distributions is to look for characterizations as processes $\{X(t) = Y(Z(t)), t \geq 0\}$, where $\{Z(t), t \geq 0\}$ is a subordinator, a positive, increasing, infinitely divisible process. In the different processes the Orthogonal polynomials are the classical Jacobi, Hermite, Laguerre and Meixner polynomials. As an example, all processes with Gamma stationary distributions and Laguerre polynomials as eigenfunctions are processes which are subordinated to the Laguerre diffusion, a branching process diffusion with immigration. The idea of subordination in Jacobi processes goes back to Bochner (1954).

Local and global existence to fractional derivatives equations depending on the behavior of initial conditions for large $|x|$

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We give some necessary conditions for local and global existence of a solution to reaction-diffusion system of type (FDS) with temporal and spatial fractional derivatives. As in the case of a single equation of type (STFE) studied in [1], we prove that these conditions depend on the behavior of initial conditions for large $|x|$.

[1] M. Kirane, Y. Laskri and N. Tatar, "Critical exponents of Fujita type for certain evolution equations and systems with spatio-temporal fractional derivatives", *J. Math. Appl.*, **312** (2005) 488-501.

Zeros of Generalized Rogers-Ramanujan Series

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Series expansions for the zeros of generalized Rogers-Ramanujan functions appear in Ramanujan's Lost Notebook. I will discuss how to obtain extensions of Ramanujan's formulas by analyzing the zero distribution of certain orthogonal polynomials. I will also mention the analytic and combinatorial significance of series which arise in this investigation.

Fractional integration of H-function of multivariables and applications

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The main object is to derive a number of key formulas for the fractional integration of the multivariable H-function. Each of the general eulerian integral formulas are shown to yield interesting new results for various families of generalized hypergeometric functions of several variables. Some of these applications of the key formulas would provide potentially useful generalizations in the theory of fractional calculus.

Gaussian rational quadrature formulas for ill-scaled integrands

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Let f be a function analytic in a neighborhood of a real interval I, and $W(x)$ be a weight function which may have some integrable singularities at the endpoints of I. We are interested in evaluating

$$\mathcal{I}_W(f) = \int_I f(x)W(x)dx,$$

by means of a Gaussian quadrature formula based on rational functions (GRQF). It is well known that the analyticity of f implies that GRQF converges to $\mathcal{I}_W(f)$ with geometric rate. Nevertheless, from the numerical point of view slow convergence is produced when the mass of the integral is heavily concentrated at some points of I, possibly due to difficult poles of f. We consider a rational modification of the type B_nW/A_n and a corresponding efficient routine which allows to deal with this problem.

Asymptotic zero distribution of some hypergeometric polynomials

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In recent papers ([1], [2]), Martínez-Finkelshtein and Kuijlaars and their co-authors have used Riemann-Hilbert methods to derive the asymptotic zero distribution of Jacobi polynomials $P_n^{(\alpha_n,\beta_n)}$ when the limits

$$A = \lim_{n\to\infty} \frac{\alpha_n}{n} \text{ and } B = \lim_{n\to\infty} \frac{\beta_n}{n}$$

exist and lie in the interior of certain specified regions in the AB-plane. We prove that the zeros of the function ${}_2F_1\left(-n, \frac{n+1}{2}; \frac{n+3}{2}; z\right)$ asymptotically approach the section of the lemniscate

$$\left\{z : \left|z(1-z)^2\right| = \frac{4}{27}; \mathrm{Re}(z) > \frac{1}{3}\right\}$$

as $n \to \infty$. Our result corresponds to one of the transitional or boundary cases for Jacobi polynomials in the Kuijlaars Martínez-Finkelshtein classification.

[1] A. B. J. Kuijlaars, and A. Martínez-Finkelshtein, "Strong asymptotics for Jacobi polynomials with varying nonstandard parameters", *J. Anal. Math.*, 94 (2004) 195-234.
[2] A. Martínez-Finkelshtein, P. Martínez-González, and R. Orive, "Zeros of Jacobi polynomials with varying non-classical parameters", *Special functions* (Hong Kong, 1999), pp 98-113, World Sci. Publishing, River Edge, NJ, 2000.

Interlacing of zeros of shifted sequences of one-parameter orthogonal polynomials

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We study the interlacing property of zeros of Laguerre polynomials of adjacent degree, where the free parameters differ by an integer, and of the same degree, where the free parameter is shifted continuously. Similar interlacing results are proven for the positive zeros of Gegenbauer polynomials.

Perturbation of multiple orthogonal polynomials

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We are interested in the asymptotic properties of the system of polynomials defined by recurrences

$$Q_{n+1} = (z - a_{n,n})Q_n - a_{n,n-1}Q_{n-1} - \ldots - a_{n,n-p}Q_{n-p}, \quad n \geq 0$$

with initial conditions $Q_j = 0,\ j < 0$. In some general cases the polynomials Q_n are known to be multiple orthogonal (Favard type theorem) with respect to a system of measures. Their asymptotic properties are closely related with spectral properties of associated band Hessenberg difference operator. We study the case of compact perturbation of background difference operator where the polynomials are defined by recurrences with constant coefficients (Joint work with A. Aptekarev and E. Saff).

Indeterminate moment problem associated with symmetric Al-Salam and Chihara polynomials

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We study an explicit indeterminate moment problem related to the Al-Salam and Chihara polynomials in base $q > 1$, and we restrict our attention to the symmetric case. This is a one-parameter family extending the continuous q^{-1}-Hermite polynomials. We give a set-up for finding orthogonality measures for the associated moment problem. Here we make essential use of the fact that these polynomials occur in the q-analogue of the Askey scheme, so that these polynomials occur as eigenfunctions of a suitable second order difference operator. This operator is self-adjoint on a suitable Hilbert space, and we give the spectral decomposition. From this decomposition we obtain the orthogonality measures recently introduced by Christiansen and Ismail (2006).

As a limit case we obtain the N-extremal measures for the moment problem associated to the continuous q^{-1}-Hermite polynomials, a result due to Ismail and Masson (1994). We discuss some additional results and applications. Moreover, a similar approach can be carried through for the case of the Stieltjes-Wigert polynomials (Joint work with Jacob S. Christiansen, to appear in Constructive Approximation).

Askey-Wilson polynomials and an embedding of Zhedanov's algebra AW(3) in a double affine Hecke algebra

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Zhedanov's algebra AW(3) will be considered with explicit structure constants such that, in the basic representation, the first generator becomes the second order q-difference operator for the Askey-Wilson polynomials. This representation is faithful for a certain quotient of AW(3) such that the Casimir operator is equal to a special constant. Some explicit aspects of the double affine Hecke algebra (DAHA) related to symmetric and non-symmetric Askey-Wilson polynomials will be presented without requiring knowledge of general DAHA theory. Finally a central extension of this quotient of AW(3) will be introduced which can be embedded in the DAHA by means of the faithful basic representations of both algebras.

T.H. Koornwinder, "The relationship between Zhedanov's algebra AW(3) and the double affine Hecke algebra in the rank one case", `arXiv:math.QA/0612730`.

On certain asymptotic expansions of multiple zeta and gamma functions

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We study the error term of certain asymptotic expansions for the multiple zeta and the logarithm of the multiple gamma functions. We establish the complete monotonicity of the remainder of even order in the asymptotic expansion of the logarithm of the double gamma function. We also derive an error bound for this expansion. Some related inequalities for multiple Bernoulli polynomials and numbers are also discussed.

Inverse scattering, asymptotic properties of reproducing kernels and canonical systems

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We are interested in asymptotic properties of reproducing kernels coming from a special functional model. The model gives rise to a class of canonical systems containing Schrödinger (Sturm-Liouville) operators. The results are then applied to the inverse scattering for these systems.

Sturmian methods

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We present some results on zeros of orthogonal polynomials obtained as applications of the Sturm comparison theorem used in different forms, chiefly that due to G. Szego. These results arise as part of our preparation of a chapter on this topic for the new "Bateman project".

Two Generalized Fractional Integrals of confluent Hypergeometric Functions with Two Variables

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We investigate the two generalized fractional integrals with confluent hypergeometric functions of two variables. We prove the theorems on fractional integral related to confluent hypergeometric function of two variables. Our result demonstrates the importance of understanding the techniques of fractional integral for confluent hypergeometric functions of two variables.

Extremal functionals for vector equilibrium problems

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We will consider analogues of Maskar-Saff extremal functionals [1] for vector equilibrium problems in the theory of logarithmic potential. These functionals attain their minimums on supports of equilibrium measures. There are such functionals for vector equilibrium problems with Nikishin interaction matrix in [2]. We are going to construct such functionals for equilibrium problems with Anjelesko interaction matrix and consider some of their applications.

[1] H.N. Maskar and E. B. Saff, "Where does the sup norm of a weighted polynomial live?", *Constr. Approx.*, **1** (1985) 71-91.
[2] M. A. Lapik, "Support of the extremal measure in a vector equilibrium problem", *Sb.Math.*, **197** : 8 (2006) 1205-1221.

Perturbed General Recurrence Relations

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We present some new results on the perturbations of the polynomials P_k defined by the general recurrence relation:

$$P_{-1} \equiv 0, \quad P_0(z) = 1, \quad \forall k \geq 0 \ P_{k+1}(z) = (z - a_k)P_k(z) - \sum_{i=max(-1,k-q)}^{k-1} b_{i+1}^{[k]} P_i(z),$$

where a_k, $b_{i+1}^{[k]}$ are complex numbers and $q \in N^* \bigcup \{\infty\}$, N^* denotes the set of the integers (> 0). More especially we study the effects on the properties of the family $\{P_k\}$ when we perturb the coefficients a_k, $b_j^{[k]}$ and the order q.

The method [2] presented here is a generalization of [1] (devoted to the three-term recurrence relations) and we give some new applications of this general method for the orthogonal polynomials and more precisely for the polynomials defined by the Sobolev inner product

$$< P, Q > := \sum_{s=0}^{m} \int_R P^{(s)}(t) \overline{Q^{(s)}}(t) d\mu_s(t).$$

[1] E.Leopold," Perturbed recurrence relations", International Conference on Numerical Algorithms, *Numerical Algorithms* **33** (2003) 357-366.
[2] E.Leopold, "Perturbed recurrence relations, II The General case", to appear in *Numerical Algorithms*.

Two-variable orthogonal polynomials of big q-Jacobi type

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A four parameter family of orthogonal polynomials in two discrete variables is defined for a weight function of basic hypergeometric type. The polynomials are expressed in terms of univariate big q-Jacobi polynomials. They form an extension of Dunkl's bivariate (little) q-Jacobi polynomials [1]. We prove orthogonality property of the new polynomials, and show that they satisfy a three-term relation in a vector matrix notation, as well as a second order partial q-difference equation. Also, we give formulas relating three bivariate families of big q-Jacobi, q-Hahn [2], and generalized Bernstein polynomials [3].

[1] C. F. Dunkl, "Orthogonal polynomials in two variables of q-Hahn and q-Jacobi type", *SIAM J. Alg. Disc. Math.*, **1** (1980) 137-151.
[2] G. Gasper and M. Rahman, "Some systems of multivariable orthogonal q-Racah polynomials", *Ramanujan J.*, **13** (2007) 389-405.
[3] S. Lewanowicz, P. Woźny, I. Area and E. Godoy, "Multivariate generalized Bernstein polynomials, and new identities for bivariate q-Hahn and q-Jacobi polynomials" (submitted).

On the group structure of Bailey's transformations for ${}_{10}\phi_9$-series

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The concept of symmetry groups associated with two term transformations for basic hypergeometric series is well known, and most of them have been studied and identified ([1] and references therein). One two term identity for which the invariance group, to our knowledge, was not written down explicitly is Bailey's four term transformation for non-terminating ${}_{10}\phi_9$-series considered as a two term transformation between a linear combination of such series which we call Φ. It is shown that the invariance group of this transformation is the Weyl group of type E_6.

We demonstrate that the group associated with a three term transformation between Φ-series, each admitting Bailey's two term transformation, is the Weyl group of type E_7. We do this by giving a description of the root system of type E_7 that allows to find a transformation between equivalent three term identities in an easy way. We show how one can find a prototype of each of the five essentially different identities. The results of this research can be found in [2].

[1] J. Van der Jeugt and K. Srinivasa Rao, "Invariance groups of transformations of basic hypergeometric series", *J. Math. Phys.*, **40**(12) (1999) 6692-6700.
[2] S. Lievens and J. Van der Jeugt, "Symmetry groups of Bailey's transformations for ${}_{10}\varphi_9$-series", *J. Comput. Appl. Math*, in press, available online, doi:10.1016/j.cam.2006.08.005

Energy and Distribution of Leja sequences on the circle

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Given a point set $\mathcal{Z}_N = \{z_k : k = 0, \ldots, N\}$, the s-energy of $\mathcal{Z}_N$ is defined as

$$E(\mathcal{Z}_N; s) = \sum_{0 \le i \ne j \le N} K(|z_i - z_j|; s),$$

where $|\cdot|$ denotes the Euclidean norm and

$$K(t; s) = \begin{cases} t^{-s}, & \text{if} \quad s > 0, \\ -\log(t), & \text{if} \quad s = 0 \end{cases}$$

is the Riesz kernel. A sequence of point sets $\{\mathcal{Z}_N\}$ is called asymptotically s-energy minimizing on the unit circle $\mathbb{T}$ (briefly, $\{\mathcal{Z}_N\} \in AEM(\mathbb{T}; s)$) if $E(\mathcal{Z}_N; s)$ behaves asymptotically like the energy of minimal energy configurations. We prove that Leja sequences do not depend on s, but they are $AEM(\mathbb{T}; s)$ for $s \le 1$, and not for $s > 1$. As a consequence of being $AEM(\mathbb{T}; s)$ for $s < 1$, Leja sequences are distributed uniformly on $\mathbb{T}$.

A very general asymptotic method for Mellin convolution integrals

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We present a new method for deriving asymptotic expansions of

$$\int_a^b f(t) h(xt) dt$$

for small x and (a, b) a bounded or unbounded interval. We only require for $f(t)$ and $h(t)$ to have asymptotic expansions at $t = \infty$ and $t = 0$ respectively. Remarkably, it is a very general technique that unifies a large set of asymptotic methods. Watson's Lemma and other classical methods, Mellin transform techniques, McClure and Wong's distributional approach and the method of analytic continuation turn out to be simple corollaries of this method. In addition, the most amazing thing about it is that its mathematics are absolutely elemental and do not involve complicated analytical tools as the aforesaid methods do: it consists of simple "sums and substractions". Many known and unknown asymptotic expansions of important integral transforms are trivially derived from the approach presented here.

Relative asymptotic of Hermite Padé orthogonal polynomials of Nikishin systems

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Let $\mathcal{N}(\sigma_1, \ldots, \sigma_m)$ and $\mathcal{N}(g_1\sigma_1, \ldots, g_m\sigma_m)$ be two Nikishin systems of measures where $\sigma_k' > 0$ a.e. on its support consisting of an interval $[a_k, b_k]$. We consider sequences of multiple orthogonal polynomials with respect to the two Nikishin systems and study the asymptotic behavior of their ratio for appropriate sequences of multi-indices and conditions on the weights $g_k, k = 1, \ldots, m$.

Quadratic decomposition of Appell sequences

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We proceed to the quadratic decomposition of Appell sequences and we prove that the four derived sequences obtained by this approach are also Appell sequences with respect to another (lowering) operator. Thus, we introduce and develop the concept of the Appell polynomial sequences with respect to that same operator. More developments, specially those concerning with the orthogonality of such sequences, will be presented.

Orthogonal Polynomials of Iterated Function Systems with uncountably many maps and their Fourier Transform

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I will describe a numerically stable technique for computing the Jacobi matrix contraction ratios. I will study analytically the nature of the associated invariant measures and of their Fourier transforms. I will prove a simple result that nonetheless extends those obtained by Peres, Simon and Solomyakon the absolute continuity of invariant measures of random IFS, that can also be treated within the proposed formalism. Finally, I will show how these techniques permit to solve, for the first time in a numerically stable fashion, an inverse fractal problem due to Elton and Yang, that might have relevance in numerical integration and in the treatment of digital images.

Dirichlet problem associated to Dunkl laplacian and applications

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We prove some potential theoretical properties of harmonic functions associated to Dunkl operators. We solve the corresponding Dirichlet problem and establish the related Harnack principle and normality criteria.

Orthogonal Polynomials and Gaussian Quadratures for an Oscillatory Weight

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We consider the necessary conditions for the existence of a sequence of polynomials $\{\pi_n\}_{n=0}^{+\infty}$ orthogonal with respect to the linear functional

$$\mathcal{L}(p) = \int_{-1}^{1} p(x) \exp(\mathrm{i}\zeta x)(1-x^2)^{\alpha} dx, \quad \alpha > -1, \quad \zeta \in \mathbb{R}, \quad p \in \mathcal{P}.$$

Using certain properties of the “oscillatory Gegenbauer weight”

$$w(x) = \exp(\mathrm{i}\zeta x)(1-x^2)^{\alpha},$$

we give some properties of the corresponding orthogonal polynomials (coefficients of the three-term recurrence relation, the differential equation for orthogonal polynomials with respect to $\mathcal{L}$, the zero distribution of $\pi_n(z)$). A numerical construction of the three-term recurrence coefficients is also considered. Finally, we discuss a possibility for constructing the Gaussian quadratures with respect to the moment functional $\mathcal{L}$, as well as some methods for proving the convergence of such Gaussian quadrature rules.

Integral Representations for Orthogonal Polynomials

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In a recent paper [1], using the Riemann-Hilbert approach to orthogonal polynomials (OP) the authors derived an asymptotic, Cauchy-type integral representation for polynomials that are orthogonal over the unit circle with respect to a positive analytic weight. Using such a representation, they were able to describe the asymptotic behavior of these polynomials and their zeros. In this talk we will show how to obtain similar results for other (more difficult) systems of OP, namely, when the orthogonality is considered over an arbitrary planar domain bounded by an analytic curve, or over the analytic curve itself. We will then use these integral representations to obtain new results on the asymptotic behavior of the polynomials, and in particular, we will be able to discern the fine structure of their zeros. An interesting comparison between OPUC and OP over the unit disk will be presented.

[1] A. Martinez-Finkelshtein, K. T. R. McLaughlin and E. B. Saff, "Szegö orthogonal polynomials with respect to an analytic weight: canonical representation and strong asymptotics", *Constr. Approx.* **24** (2006) 319-363.

Discrete Painlevé and Garnier transcendents from similarity reduction on the lattice

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In his seminal paper of 1912 René Garnier introduced a system of coupled PDEs arising from the isomonodromic deformation problem of a linear second order differential equation with multiple moving "essential" singularities. This system contains in particular a subsystem of higher order ODEs constituting the higher-order analogues of the celebrated Painlevé VI equation. Whereas the construction of discrete analogues of the Painlevé equations (i.e. of nonlinear non-autonomouns ordinary difference equations possessing similar properties) have formed a thriving subject of research over the past decade, discrete analogues of the Garnier system have not been considered widely. In this talk we will present a symmetry approach for obtaining such systems of difference equations, namely by performing a similarity reduction from integrable lattice equations. Also a q-analogue of this approach will be considered leading to higher-order higher-degree q-difference equations which are expected to be closely related to some recently contructed q-Garnier systems by H. Sakai.

[1] R. Garnier, "Sur des équations du troisième ordre dont l'intégrale générale est uniforme et sur une classe d'équations nouvelles d'ordre supérieur", *Ann. École. Norm. Sup.*, **29** (1912) 1-126.
[2]F.W. Nijhoff and A.J. Walker, "The discrete and continuous Painleve VI hierarchy and the Garnier systems", *Glasgow Math. J.*, **43A** (2001) 109-123.
[3] A. Tongas and F.W. Nijhoff, "A discrete Garnier type system from symmetry reduction on the lattice", *J. Phys. A: Math. gen.*, **39** (2006) 12191-12202.

Logarithmic order and type of indeterminate moment problems

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For an indeterminate moment problem the four entire functions appearing in the Nevanlinna parametrisation of all solutions to the moment problem are known to have a common order and type.

Several indeterminate moment problems of common order 0 have been investigated. The growth of the entire functions in these situations can be investigated using a refined scale, called logarithmic order and type. The purpose of this talk is to give examples of indeterminate Stieltjes moment problems and thereby to demonstrate that there exist indeterminate moment problems of any prescribed logarithmic order and type (Joint work with Christian Berg).

A simplified version of the Laplace's method with applications to special functions

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We revise the standard Laplace's method of asymptotic expansions of integrals of the form

$$F(x) = \int_0^b e^{-xf(t)} g(t)\, dt,$$

for large values of x. The standard method requires a change of variable which, in general, makes the computation of the coefficients of the expansion very complicated. Usually, only the first few terms of the expansion can be computed explicitly. One century ago, Burkhardt and Perron suggested a modification of the method based on an expansion of both functions $f(t)$ and $g(t)$, at the neighbourhoods of the critical points of the phase function $f(t)$ (see [1], chapter 2). We explore this idea and propose a variant of the method which avoids that change of variable and simplifies the computations. With this modification of the Laplace's method, the calculation of the coefficients of the asymptotic expansion is remarkably simpler. On the other hand, the asymptotic sequence is as simple as in the standard Laplace's method: inverse powers of the asymptotic variable. As an illustration, we apply this modified Laplace's method to two important special functions: the Gamma function $\Gamma(z)$ foe large z and the Gauss hypergeometric function ${}_2F_1(a, b; c; z)$ for large b and c, obtaining new asymptotic expansions of these functions. Also, as a consequence, we obtain an explicit formula for the coefficients of the classical Stirling expansion of $\Gamma(z)$ for large z.

[1] A. Erdélyi, *Asymptotic expansions*, Dover Publications Inc., New-York, (1956).

Zero and logarithmic asymptotics of contracted Sobolev orthogonal polynomials for exponential weights

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We study the asymptotic behavior of the zeros of a sequence of polynomials whose weighted norms, with respect to a sequence of weight functions, have the same nth root behavior as the weighted norms of certain extremal polynomials. Our result is applied to obtain the (contracted) weak zero asymptotics for orthogonal polynomials with respect to a Sobolev inner product with exponential weights.

Krall type orthogonal polynomials in several variables

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In this work, we study orthogonal polynomials in several variables associated with a positive measure μ consisting of an absolutely continuous part μ_c perturbed by the addition of one or more Dirac masses. Our main objective is to study the connection between orthogonal polynomials associated with measure μ and those associated with μ_c. In particular, differential and explicit expressions relating both orthogonal families of polynomials are obtained. Finally, some examples are given.

Error estimates for Gaussian quadratures of analytic functions

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For analytic functions the remainder term of Gaussian quadrature formula and its Kronrod extension can be represented as a contour integral with a complex kernel. We study these kernels on elliptic contours with foci at the points ± 1 and a sum of semi-axes $\rho > 1$ for the Chebyshev weight functions of the first, second and third kind, and derive representation of their difference. Using this representation and following Kronrod's method of obtaining a practical error estimate in numerical integration, we derive the new error estimates for Gaussian quadratures and compare them with the bounds from [1].

[1] W. Gautschi and R. S. Varga, "Error bounds for Gaussian quadratures of analytic functions", *SIAM J. Numer. Anal.*, **20** (1983) 1170-1186.

Probabilistic origin of a 2-variable orthogonal polynomial system

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We present a probabilistic approach to the generation of a (possibly) new system of orthogonal polynomials in 2 variables that generalizes the Krawtchouk polynomials. This extends our earlier work on the "Cumulative Bernoulli Processes" that we introduced in 1983, where the probability transition kernel was shown to have the Krawtchouk polynomials as their eigenfunctions. A brief description of the 2-dimensional cumulative Bernoulli process is given and a transition kernel is formed by a convolution of the binomial and trinomial distributions. The eigenfunctions of this kernel turn out to be this new system, which is obtained as a special limiting case of the 9-j symbols that appear in the quantum angular momentum theory.

Concentration of the integral of idempotent exponential polynomials

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We prove that for all $p > 1/2$ there exists a constant $c_p > 0$ such that, for any symmetric measurable set of positive measure $E \subset \mathbb{T}$ and for any $c < c_p$, there is an idempotent trigonometrical polynomial f satisfying $\int_E |f|^p > c \int_{\mathbb{T}} |f|^p$. This disproves a conjecture of Anderson, Ash, Jones, Rider and Saffari, who proved the existence of $c_p > 0$ for $p > 1$ and conjectured that it does not exists for $p = 1$.

Furthermore, we prove that one can take $c_p = 1$ when $p > 1$ is not an even integer, and that concentrating idempotent polynomials can be chosen with arbitrarily large gaps when $p \neq 2$. This shows striking differences with the case $p = 2$, for which the best constant is strictly smaller than 1/2, as it has been known for twenty years, and for which having arbitrarily large gaps with such concentration of the integral is not possible, according to classical theorems of Wiener and Ingham.

We find sharper results for $0 < p \leq 1$ when we restrict to open sets, or when we enlarge the class of idempotent trigonometric polynomials to all positive definite ones. Our results essentially improve all know estimates of the concentration constants for all $p(\neq 2)$. We also explore connections to the Hardy-Littlewood majorant problem, including a recent solution to Montgomery's conjecture by Mockenhaupt and Schlag. Our work employs several techniques which are of independent interest.

Finite-type relations for monic orthogonal polynomials in two discrete variables

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A theory of finite-type relations between polynomial sequences in the univariate case was developed in [1]. In our earlier works [2], [3] we systematically studied the orthogonal polynomial solutions of a second order partial difference equation of hypergeometric type of two variables.

In this talk we present an explicit difference-derivative representation and structure relations in vector-matrix form for monic orthogonal polynomials of hypergeometric type in two discrete variables. This approach is discussed in detail for bivariate Hahn, Meixner, Kravchuk and Charlier polynomials.

[1] P. Maroni, "Semi-classical character of finite-type relations between polynomial sequences", *Appl. Num. Math.*, **31** (1999) 295-330.
[2] J. Rodal, I. Area and E. Godoy, "Orthogonal polynomials of two discrete variables on the simplex", *Integral Transforms and Special Functions*, **16** (3) (2005) 263-280.
[3] J. Rodal, I. Area and E. Godoy, "Linear partial difference equations of hypergeometric type: orthogonal polynomial solutions in two discrete variables", *J. of Comput. Appl. Math.*, **200** (2007) 722-748.

Extension to confluent Heun's equations of some properties of the the General Heun Equation

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It is well known(Kuiken, Maier [1]) that the General Heun Equation(GHE) is linked to the Hypergeometric

equation by peculiar rational transformations of the independent variable. On the other hand, derivative (DHeun) of solutions of the GHE can also, in some cases [2], be written as other GHE with modified parameters,noticing that this appears too for solutions of the Hypergeometic equation but in all cases. Our work [3] shows that these 2 transformations: Hypergeometric to Heun and DHeun to Heun extend with some constraints, to the 4 confluent families of the GHE: the CHE- Confluent Heun Equation, the DHE-Double Confluent Heun Equation, the BHE-Biconfluent Heun Equation, the THE-Triconfluent Heun Equation.

[1] R. S. Maier, *J.Differential Equations*, **215** (2005) 171-203.
[2] A. Isjkanyan and K. A. Suominen, J.Phys.A: Math.Gen. **36** (2003) L81-85.
[3] A. Aderibighe, M. N. Hounkonnou and A. Ronveaux, Preprint ICMPA (2007).

Fisher informations of special functions from their second-order differential equations

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The information theory of orthogonal polynomials is extended to other special functions of applied mathematics and mathematical physics, provided they are solutions of second-order differential equations. In this communication we will show explicit expressions for various information-theoretic properties by means of the expectation values of the coefficients of the differential equations. Emphasis will be made in the Fisher information and relative Fisher information for the Rakhmanov probability distributions obtained by squaring the involved special functions. We illustrate our approach for various specific special functions of physico-mathematical interest (e.g., Airy, generalized hypergeometric-type functions, non-relativistic wavefunctions).

2-index Clifford-Hermite polynomials with applications in wavelet analysis

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Clifford analysis (see for e.g. [1] and [2]), centred around the notion of monogenic function, may be regarded as a direct and elegant generalization to higher dimension of the theory of holomorphic functions in the complex plane. It has proven to be an appropriate framework for considering higher dimensional continuous wavelet transforms, the construction of the wavelets being based on specific types of higher dimensional orthogonal polynomials, such as the Clifford-Hermite polynomials. These polynomials, introduced by Sommen in [3] as a Clifford generalization of the classical Hermite polynomials on the real line, are indeed the desired building blocks for the so-called Clifford-Hermite wavelets which offer a refinement of the traditional Marr wavelets (see [4]). In this contribution, a generalization of the Clifford-Hermite polynomials to a 2-parameter family is introduced by taking the double monogenic extension of a modulated Gaussian, i.e. the classical Morlet wavelet. The ultimate goal being the construction of new Clifford-wavelets refining the traditional Morlet wavelet, we first thoroughly investigate the properties of the newly introduced polynomials, amongst which are their recurrence and orthogonality relations and the transformation formulae between the different types of Clifford-Hermite polynomials (Joint work with Fred Brackx, Hennie De Schepper and Frank Sommen).

[1] F. Brackx, R. Delanghe and F. Sommen, *Clifford Analysis*, Pitman Publ., Boston-London-Melbourne, 1982.
[2] F. Sommen, "Special functions in Clifford analysis and axial symmetry", *J. Math. Anal. Appl.* **130**, No. 1 (1988), 110-133.
[3] F. Brackx and F. Sommen, "Clifford-Hermite Wavelets in Euclidean Space", *The Journal of Fourier Analysis and Applications* **6**, No. 3 (2000) 299-310.

Spectral Analysis of Immigration-Emigration Processes with Linear and q-Linear Killing Rates

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A population process with constant birth rates $\lambda_n \equiv \lambda$ and death rates $\mu_n \equiv \mu$ is supplied with strong linear killing rates $\gamma_n = \gamma n$ for $n \in \mathbb{N}_0$. The process is analyzed in view of its spectral representation: The underlying orthogonal polynomials are seen to be Lommel polynomials $R_{n,\nu}(x)$, considered as functions in their parameter ν. Computing the Stieltjes transform of their orthogonality measure, we recognize that it is purely discrete and given by the zeros of a Bessel function $J_{\nu-1}(x)$ in its order, in agreement with earlier results due to Maki. Qualitative and quantitative results for the zeros are worked out via methods from the theory of Bessel functions and from regular perturbation theory. Quite naturally a basic extension of the process with bounded q-linear killing rates is discussed. Some qualitative analysis shows that its spectrum consists of an absolutely continuous part and at most finitely many mass points.

Irrationality proof of certain Lambert series using little q-Jacobi polynomials

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We apply the Padé technique to find rational approximations to

$$h^{\pm}(q_1, q_2) = \sum_{k=1}^{\infty} \frac{q_1^k}{1 \pm q_2^k}, \qquad 0 < q_1, q_2 < 1, \quad q_1 \in \mathbb{Q}, \quad q_2 = 1/p_2, \quad p_2 \in \mathbb{N} \setminus \{1\}.$$

A separate section is dedicated to the special case $q_i = q^{r_i}, r_i \in \mathbb{N}, q = 1/p, p \in \mathbb{N} \setminus \{1\}$. In this construction we make use of little q-Jacobi polynomials. Our rational approximations are good enough to prove the irrationality of $h^{\pm}(q_1, q_2)$ and give an upper bound for the irrationality measure.

Unit circle semiclassical functionals belonging to the class (p, p)

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This paper deals with the study the semiclassical linear functionals u that verify an equation as follows:

$$(*) \quad D(A(z)u) = \Big(\lambda_p \Phi_p(z)\Big)u, \quad \lambda_p \in \mathbb{C} - \{0\},$$

where Φ_p denotes the fixed term of degree p of the orthogonal polynomial sequence (OPS), $\{\Phi_n\}$, associated to u and D means the derivative operator. Also, we assume that the polynomial A is self-reciprocal of degree p.

The proposed problem is a generalization to degree p of the equation (*) verified by the semiclassical Jacobi family for $p = 2$. In the development of the solution, in a natural way, the equation (*) leads to several cases for the functional u. Here, we solve one of them. For this case, we find a homogeneous difference equation for the sequence of the moments corresponding to u, $\{u_n\}$, i.e. $u_n = u(z^n)$. Moreover, we give the terms of the (OPS), $\{\Phi_{p+n}\}$, as the solution of a homogeneous second order differential equation.

Positive definiteness with missing data

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I intend to exhibit a comprehensible version of my joint work (still in progress) with Dariusz Cichoń and Jan Stochel on extending positive definiteness from not-too-complete date to a richer structure so as to get an integral representation. The whole story in a sense is in the spirit of

[1] J. Stochel and F. H. Szafraniec, "The complex moment problem and subnormality: a polar decomposition approach", *J. Funct. Anal.* **159** (1998) 432-491,

[2] J. Stochel and F. H. Szafraniec, "Unitary dilation of several contractions", *Operator Theory: Adv. Appl.* **127**, Birkhäuser, Basel 2001, 585-598.

Tridiagonal pairs and the q-tetrahedron algebra

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Let $\mathbb{K}$ denote a field and let V denote a vector space over $\mathbb{K}$ with finite positive dimension. By a *Leonard pair* on V we mean an ordered pair of linear transformations $A : V \to V$ and $A^* : V \to V$ that satisfy the following:

1. There exists a basis for V with respect to which the matrix representing A is diagonal and the matrix representing A^* is irreducible tridiagonal;
2. There exists a basis for V with respect to which the matrix representing A^* is diagonal and the matrix representing A is irreducible tridiagonal.

It is known that the Leonard pairs are in bijection with the orthogonal polynomials from the terminating branch of the Askey scheme. This branch includes the q-Racah polynomials and their relatives. In this talk we consider a mild generalization of a Leonard pair called a *tridiagonal pair*. We also consider the q-tetrahedron algebra, which can be viewed as a q-analog of the three-point $\mathfrak{sl}_2$ loop algebra. We obtain a tridiagonal pair from each finite-dimensional irreducible module for the q-tetrahedron algebra.

On the asymptotic behaviour of recurrence coefficients for orthogonal polynomials with exponential weight

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We will consider orthogonal polynomials $\{p_{n,N}(x)\}_{n=0}^{\infty}$ on the real line with respect to a weight $w(x) = e^{-NV(x)}$ and in particular the asymptotic behavior of the corresponding recurrence coefficients.

For specific $V(x)$ it is known that the recurrence coefficients converge for $N = n$ to some limit at a rate of $\mathcal{O}\left(\frac{1}{n^2}\right)$. During this talk we will show, using the Deift-Zhou method of steepest descent for Riemann-Hilbert problems, that this result is in fact always true for the one-cut regular case.

Continued fraction method of an estimation of a Stieltjes series with complex coefficients

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The aim of this work is to estimate a Stieltjes function

$$f_1(z) = \int_0^1 \frac{d\gamma_1(u)}{1+zu}, \quad z \in \mathbb{C}\backslash(-\infty,-1), \quad d\gamma_1(u) \geq 0, \quad f_1(-1) = 1 \tag{5}$$

from the Taylor expansions of $f_1(z)$ available at several complex points $z = z_j \in \mathbb{C}\backslash(-\infty,-1)$ and at one real one $z = -1$. To solve the problem formulated we use the method of linear fractional transformation interrelated $f_1(z)$ with $f_{N+1}(z)$ as follows (cf. (5)):

$$zf_1(z) = z_1 f_1(z_1) + \frac{f_1(z_1)(z-z_1)}{1+\theta_2 z f_2(z)}, \; \ldots \; zf_N(z) = z_N f_N(z_N) + \frac{f_N(z_N)(z-z_N)}{1+\theta_{N+1} z f_{N+1}(z)}. \tag{6}$$

The recurrent relations (6) lead to the following multipoint continued fraction

$$zf_1(z) = z_1 f_1(z_1) + \cfrac{f_1(z_1)(z-z_1)}{1+\theta_2\left(z_2 f_2(z_2) + \cfrac{f_2(z_2)(z-z_2)}{1+\theta_3\left(\cdots + \cfrac{\cdots}{1+\theta_{N+1} z f_{N+1}(z)}\right)}\right)}. \tag{7}$$

Here the complex coefficients θ_{j+1} are chosen in such a way that $f_j(-1) = 1$, $j = 1, 2, ..., N+1$. It is proved that, if $f_1(z)$ is a Stieltjes function then $f_{N+1}(z)$ appearing in (7) is a Stieltjes function as well (cf. 5). Since the elementary estimate of a function $f_{N+1}(z)$ is available [2, pp. 41. eq. 1.154] then the general one of $f_1(z)$ is also available. It follows directly from (7).

The Stieltjes functions $f_1(z)$ with restrictions given in (5) represent, among many others, the effective coefficients of inhomogeneous media [2,3]. Hence the results obtained are useful for material engineering especially for designing the optimal properties of composite materials. Nontrivial practical evaluations are also provided.

[1] G. A. Baker, Jr. and P. Graves-Morris, “Padé Approximants”, Cambridge University Press, London, 1996.
[2] J. Gilewicz, M. Pindor, J. J. Telega, and S. Tokarzewski., “N-point Padé approximants and two sided estimates of errors on the real axis for Stieltjes functions”, *J. Comp. Appl. Math.*, **178** (2005) 247-253.
[3] S. Tokarzewski, “Multipoint continued fraction approach to the bounds on effective transport coefficients of two-phase media”, *IFTR Reports*, **4** (2005) 3-171.

Decomposition formulas for Srivastava's hypergeometric $H(A)$ function on Saran functions

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With the help of some techniques based upon certain inverse pairs of symbolic operators, we investigate several decomposition formulas associated with Srivastava's hypergeometric function $H(A)$ in three variables. Many operator identities involving these pairs of symbolic operators are first constructed for this purpose. By means of these operator identities, as many as ten composition formulas are then found, which are expressed through aforementioned triple hypergeometric functions in terms of Saran's hypergeometric functions.

Some new spectral properties of orthogonal polynomials

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The decay rate of a Markov process is an important quantity that characterises the speed of convergence of the time-dependent state probabilities to their limiting values. In the specific setting of a birth-death process the decay rate can be identified with the smallest point, or smallest point but one, in the support of the spectral measure of the process. The latter is the orthogonalising measure for a sequence of polynomials satisfying a three-terms recurrence relation with coefficients that are determined by the parameters of the birth-death process.

During the last decade several new properties (bounds and positivity criteria) of the decay rate of a birth-death process have been obtained by exploiting techniques that do not involve orthogonal polynomials. I will show how these results can be translated to yield new information on the smallest and largest zeros of orthogonal polynomials, and on the support of the orthogonalising measure for an orthogonal polynomial sequence, in terms of the coefficients in the three-terms recurrence relation.

The surprising almost everywhere convergence of Fourier-Neumann series

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Let J_μ denote the Bessel funtion of the first kind and order μ. It is well-known that, for $\alpha \geq -1/2$, we have

$$\int_0^\infty J_{\alpha+2n+1}(x) J_{\alpha+2m+1}(x)\,\frac{dx}{x} = \frac{\delta_{nm}}{2(2n+\alpha+1)}, \quad n,m = 0,1,2,\dots$$

Then, the system

$$j_n^\alpha(x) = \sqrt{2(\alpha+2n+1)}\, J_{\alpha+2n+1}(x) x^{-\alpha-1}, \qquad n = 0,1,2,\dots$$

is orthonormal in $L^2((0,\infty), d\mu_\alpha)$, with $d\mu_\alpha(x) = x^{2\alpha+1}\,dx$. For each suitable function f, let $S_n^\alpha f$ be the n-th partial sum of its Fourier series with respect to the system $\{j_n^\alpha\}_{n=0}^\infty$, which are usually called Fourier-Neumann series.

In the talk we study the almost everywhere convergence of Fourier-Neumann series $S_n^\alpha f$ for functions $f \in L^p((0,\infty), d\mu_\alpha)$ (with $1 < p < \infty$).

Orthogonal rational functions and spectral theory

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A central problem in the theory of orthogonal polynomials is the search for relations between the orthogonality measure and the parameters of the related recurrence relation. In particular, in the case of measures supported on the real line, an approach to this problem is possible throughout the spectral analysis of the Jacobi matrix associated with the corresponding 3-term recurrence relation. The analogue of the Jacobi matrix for the case of measures on the unit circle was only recently discovered, and it is a five-diagonal unitary matrix which provides a similar connection between the orthogonality measure and the parameters of the recurrence for the orthogonal polynomials.

The orthogonal polynomials are a particular case of a more general class of orthogonal functions of interest for many pure and applied sciences: the orthogonal rational functions with prescribed poles. In this talk it will be shown that the "matrix connection" between the orthogonality measure and the recurrence for orthogonal polynomials can be generalized to orthogonal rational functions. Such a connection relates now the measure, not only to the parameters of the recurrence, but also to the location of the prescribed poles. The corresponding matrix is the result of applying a linear fractional transformation with matrix coefficients to the five-diagonal unitary matrices of the polynomial case. In spite of its greater complexity, it will be shown that this rational "matrix connection" can be used to generalize well known results for orthogonal polynomials to the rational case.

Reduction of the Gibbs phenomenon for smooth functions with jumps by the ϵ-algorithm

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Let $f : [-\pi, \pi] \mapsto \mathbb{R}$ be a function with jumps given by its Fourier series. In order to reduce the Gibbs phenomenon of the sequence of partial sums

$$S_n(f)(t) = \frac{a_0}{2} + \sum_{j=1}^{n} [a_j \cos(jt) + b_j \sin(jt)],$$

the following procedure has been proposed (see for instance C. Brezinski): compute

$$G_n(f)(e^{it}) = S_n(f)(t) + i\widetilde{S}_n(f)(t), \text{ où } \widetilde{S}_n(f)(t) = \sum_{j=1}^{n} [a_j \sin(jt) - b_j \cos(jt)],$$

and apply the ϵ-algorithm to the sequence of partial sums of the power series

$$G(f)(z) = \sum_{j=0}^{\infty} c_j(f) z^j, \quad c_0(f) = \frac{a_0}{2}, \quad \text{et pour } j > 1, \quad c_j(f) = a_j - ib_j.$$

We then use the real part of the quantities $\epsilon_{2k}^{(n)}(t)$ to approach $f(t) = \Re(G(f)(e^{it})$. Numerical results showed an important acceleration of convergence of the sequence of partial sums and a reduction of the Gibbs phenomenon, which motivate us to give a theoretical explanation.

In this talk, we obtain bounds for the error $(f(t) - \epsilon_{2k}^{(n)}(t))$ for functions of the form $f = f_1 + f_2$, where f_1 presents some known discontinuities and f_2 has Fourier coefficients decreasing "sufficiently" fast. For such kind of functions, Fourier series converges slowly and presents oscillations near the singularities of f_1. We will show that the acceleration properties of the ϵ-algorithm depend essentially of f_1. More precisely, we consider the case of $G(f_1)$ belonging to a class of hypergeometric functions,

$$G^{(\alpha,\beta)}(z) = {}_2F_1\left(\begin{matrix} \alpha+1, 1 \\ \alpha+\beta+2 \end{matrix} \middle| z \right), \quad \text{where} \quad {}_2F_1\left(\begin{matrix} a, b \\ c \end{matrix} \middle| z \right) = \sum_{j=0}^{\infty} \frac{(a)_j\,(b)_j}{(c)_j\, j!} z^j.$$

To obtain error estimations, we use the link between the complex ϵ-algorithm and Padé approximants and we study the speed of convergence of the columns of the Padé table corresponding to Stieltjes functions and perturbed Stieltjes functions.

We will give some numerical examples, and a link with Padé-Tchebychev approximants for Tchebyshev series and other rational approximants for Fourier series.

Generalized Bounded Variation and Insertion of Point Masses

Manwah Lilian WONG

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Let $d\mu_0$ be a probability measure on the unit circle with ℓ^2 Verblunsky coefficients $(\alpha_n(d\mu_0))_{n=0}^{\infty}$ of bounded variation. We add m pure points $(z_j)_{j=1}^m$ with weights $(\gamma_j)_{j=1}^m$ to $d\mu_0$, rescale and form the probability measure $d\mu_m$. We prove that the Verblunsky coefficients of $d\mu_m$ are in the form $\alpha_n(d\mu_0) + \sum_{j=1}^m \frac{z_j^n c_j}{n} + E_n$, where the c_j's are constants of norm 1 independent of the weights of the pure points and independent of n; the error term E_n is in the order of $o(1/n)$. Furthermore, we prove that $d\mu_m$ is of $(m+1)$-generalized bounded variation, a new notion that we shall introduce in this paper. Then we use this fact to prove that $\lim_{n\to\infty} \Phi_n^*(z)$ is continuous when $z \neq 1, z_1, z_2, \ldots, z_m$ and is equal to $D(z)^{-1}$, where $D(z)$ is the Szegő function.

Strong Asymptotics for non-Hermitian Orthogonal Polynomials on a Segment

L. BARATCHART and M. YATTSELEV

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We consider strong Szegő-type asymptotics for polynomials orthogonal with respect to varying complex measures on a segment, i.e., for polynomials satisfying

$$\int_{[a,b]} t^j q_n(t) \frac{s(t)}{v_{2n}(t)} \frac{dt}{\sqrt{(t-a)(b-t)}} = 0, \quad j = 0, \ldots, n-1,$$

where v_{2n} is a polynomial of degree at most $2n$ with “nearly” conjugate-symmetric zeros in $\mathbb{C} \setminus [a,b]$ and s is a Dini-continuous complex-valued function on $[a,b]$ that may vanish in a controlled manner at a finite number of points.

Hankel operators on Fock type spaces

Hélène BOMMIER HATO and El Hassan YOUSSFI

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Let $\mathcal{A}^2(\mu)$ be Fock type space consisting of those holomorphic functions which are square integrable with respect to a rotation invariant measure μ having moments of all orders on $\mathbb{C}^n$. We consider Hankel operators $H_{\bar{f}}$ with antiholomorphic symbol $\bar{f}$ on $\mathcal{A}^2(\mu)$. We characterize boundeness, compactness and Schatten class membership of $H_{\bar{f}}$ in terms of the moments of the measure μ.

CMV matrices with asymptotically constant coefficients.
Szegö over Blaschke class, Scattering Theory

Peter YUDYTSKII

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We build the scattering theory for CMV matrices with asymptotically constant coefficients. In particular, necessary and sufficient condition for the uniqueness of inverse scattering with bounded transformation operators are given in terms of the spectral data. The related asymptotics for orthogonal polynomials on the unit circle are presented (Joint work with F. Peherstorfer and A. Volberg).

Classical orthogonal polynomials at any point: computational aspects

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Computation of $P_n(x) = \sum_{k=0}^n a_{n,k} x^k$ at any point can be obtained in several well known ways. When $y = P_n(x)$ is solution of hypergeometric equations $s(x)\, y'' + t(x)\, y' + \lambda_n\, y = 0$ (like for Classical Orthogonal Polynomials), the general technique described here (even elementary) simplifies these computations, avoiding the explicit dependence of the, a priori, 5 parameters in $a_{n,k}$. The approach used, let us call as in [1] ”generic”, gives the value of $P_n(c)$ for any c from the value of $P_n(a)$, such that $s(a) = 0$. First, using recurrently the differential equation, the formula giving $P_n(a)$ is easily proved, as well as extensions to the Delta discrete and the q-discrete cases. Next, appropriate

Mac-Laurin formulas allow to compute $P_n(c)$. Further formulas are also proposed for the rth derivatives (including Delta and q-derivatives) and also for the first associate of $P_n(x)$.

[1] W. Koepf and M. Mashed-Jamei, *J. Comp. Appl. Math.*, **191** (2006) 98-105.

Some properties of Jost functions and Weyl solutions associated with finite gap Jacobi matrices

Maxim ZINCHENKO

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In this talk several results of joint work with Barry Simon and Jacob Christiansen will be presented. In particular, properties of Jost functions and Weyl solutions will be discussed.

Hypergeometric series and approximations of mathematical constant

Wadim ZUDILIN

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I will discuss some identities for generalized hypergeometric series that were discovered quite recently in connection with rational approximations to π, π^2 and π^4. A curious thing is that most of these identities have “automatic” proofs (using creative telescoping), and a problem is to provide“human” proofs by means of classical hypergeometric summation and transformation formulas.